AEROPLANE PERFORMANCE CALCULATIONS

The Directly-Useful **D.U.** Technical Series

FOUNDED BY THE LATE WILFRID J. LINEHAM, B.Sc., M.Inst.C.E.

AEROPLANE PERFORMANCE CALCULATIONS

BY

HARRIS BOOTH

B.A., ASSOC.M.INST.C.E., F.R.AË.S.

Late Technical Adviser to the Air Department of the Admiralty
Technical Adviser to the Commercial Aeroplane Wing Syndicate

LONDON

CHAPMAN & HALL, LTD.

11 HENRIETTA ST., W.C. 2

1921

ABERDEEN: THE UNIVERSITY PRESS

EDITORIAL NOTE

THE DIRECTLY-USEFUL TECHNICAL SERIES requires a few words by way of introduction. Technical books of the past have arranged themselves largely under two sections: the Theoretical and the Practical. Theoretical books have been written more for the training of college students than for the supply of information to men in practice, and have been greatly filled with descriptions of an academic character. Practical books have often sought the other extreme, omitting the scientific basis upon which all good practice is built, whether discernible or not. The present series is intended to occupy a midway position. The information, the investigations, and the problems are to be of a directly-useful character, but must at the same time be wedded to that proper amount of scientific explanation which alone will satisfy the inquiring mind. We shall thus appeal to all technical people throughout the land, either students or those in actual practice.

AUTHOR'S PREFACE

THE science of Aeroplane Performance Calculations has become extremely important with the rapid advance of Aeronautics and has been developed to a very high degree. The Author has experienced considerable difficulty, however, in finding any detailed literature on the subject, and it is for this reason that he has written this book in the hope that Aeronautical Engineers and Designers will find it useful in supplying their special need for a practical and up-to-date handbook. As Designers will be aware, the restricted information published up to the present time has scarcely touched the fringe of the subject, and it is hoped that the readers of this book will find it to be a comprehensive and efficient instrument for saving time, since Aeroplane Performance Calculations can be very laborious if a clear and systematic method is not used.

The Author has given a considerable amount of thought and attention to the arrangement of the subject matter, and has divided it into three parts, namely:—

Part I. Descriptive and Theoretical.
Part II. Practical Procedure.
Part III. Illustrative Examples.

The Aeronautical Engineer, using the volume as a handbook in his office, will find Part II. of greatest service to him, and for this reason all the formulæ, data, and curves required are collected in this part, which, being in the middle, is easier to keep lying open on the desk whilst in use than the end sections would be. This centre part also contains directions for use of formulæ, and forms of tables for use in calculating—a feature which will be found a great time-saver.

In case any process given in Part II. is difficult to follow, reference should be made to Part III., which consists of numerical examples worked out at length. Doubt as to the validity of any formula in Part II. can readily be dispersed by reference to Part I., which contains the mathematical proofs.

Students, however, would be better advised to study Part I. very carefully, before using other sections, in order to obtain a clear idea of the theory upon which the later work is based.

The last chapter of the book contains complete performance calculations for one machine; this shows the application of the methods better than do individual examples, and, moreover, gives an idea of the amount of work usually involved in a complete set of calculations. It also indicates the order in which these calculations generally have to be taken in practice.

As far as is feasible and reasonable the technical terms and symbols laid down by the Royal Aeronautical Society in their Glossary have been employed. An exception is the use of the term "propeller" instead of "airscrew," following the lead of Mr. H. C. Watts. Other exceptions are chiefly due to the fact that the Glossary does not cover nearly all the ground of the present book.

Acknowledgments are due to Messrs. Harold Bolas, H. C. Watts, and G. E. Petty for permission to make use of their work; as for the numerous test results, of which use has been made, these form part of the great debt owed by the Aeronautical Engineer to the National Physical Laboratory. The Author also wishes to take this opportunity of tendering his grateful thanks to Mr. G. E. Petty for the invaluable assistance he has given by verifying the whole of the mathematical and numerical work.

It is hoped that few errors exist in the book, but an indication of necessary corrections or suggestions for improvement in any future edition would be welcomed.

HARRIS BOOTH.

LONDON, *March*, 1921.

ERRATA.

The reader is requested to make the following corrections and then destroy this slip.

Page 45, line 20, *for* "derived" *read* "desired".
Page 65, line 4 from bottom, *for* "altitude" *read* "attitude".
Page 117, line 13, *for* "k_{max}S" *read* "k_{Lmax}S".
Page 207, *for* "Tests on model wings, **78**" *read* "Tests on model wings, **87**".
Page 207, *for* "Wire, stream-line, 9, *150*" *read* "Wire, stream-line, 9, **82,** *150*".

CONTENTS

CHAPTER V.

CHAPTER VI.

CHAPTER VII.

PART II.

PRACTICAL PROCEDURE.

CHAPTER VIII.

CHAPTER IX.

CHAPTER X.

CHAPTER XI.

CHAPTER XII.

CHAPTER XIII.

CHAPTER XIV.

PART III.

ILLUSTRATIVE EXAMPLES.

CHAPTER XV.

CHAPTER XVI.

CHAPTER XVII.

CHAPTER XVIII.

CHAPTER XIX.

CHAPTER XX.

CHAPTER XXI.

CHAPTER XXII.

LIST OF SYMBOLS USED IN PART II

THE following symbols occur frequently in Part II. and are then *always* used with the meanings given below. When used in Part I., however, *they have not invariably the meanings here given:* such cases, however, are clear from the context.

Other symbols used in Part II. are not always of invariable meaning, but they are defined as they occur and no trouble should result.

L/D . . .	$\frac{\text{Lift}}{\text{Drag}}$.
P . . .	Horse-power required for horizontal flight.
P′ . . .	Ditto at an altitude.
P_P . . .	Whichever is the less of P_T and P_R.
P_P' . . .	Whichever is the less of P_T' and P_R'.
P_R . . .	Propeller horse-power at full revolutions.
P_R' . . .	Ditto at an altitude.
P_T . . .	Propeller horse-power with throttle full open.
P_T' . . .	Ditto at an altitude.
R . . .	Body resistance in pounds at 100 m.p.h.
R_1 . . .	Part of R in slip-stream.
R_2 . . .	Part of R outside slip-stream.
S . . .	Wing area in square feet.
S′ . . .	Part of S in slip-stream.
V . . .	Speed in m.p.h.
V′ . . .	Ditto at an altitude.
W . . .	Total weight of machine in pounds: it is the same as W_0 except in cruising calculations.
W_0 . . .	Starting total weight of machine in pounds.
d . . .	Propeller diameter in inches.
k_C . . .	Centre of pressure coefficient.
k_L . . .	Lift coefficient.
$k_{L\,max.}$ . .	Maximum value of k_L.
p . . .	·161 for stationary engines and ·262 for rotary engines.
q . . .	·839 for stationary engines and ·738 for rotary engines.
λ . . .	$k_L/k_{L\,max.}$
σ . . .	Density of air relative to standard.
σ_1 . . .	$(\sigma - p)/q$.

PART I.

DESCRIPTIVE AND THEORETICAL.

CHAPTER I.

BODY RESISTANCE.

General.—The total body resistance of a machine can be briefly defined as the force resisting the passage through the air of the complete machine with the exception of the main planes and the propeller.

This total body resistance is made up of terms due to various parts of the machine, all of which will be investigated in detail presently. Each of these terms is affected in value by the relative speed of travel through the air (generally referred to briefly as the "speed") and by the direction in which the relative air stream meets that particular part of the machine to which the term in question refers.

It may be stated at once as a general approximation that the resistance of any part of the machine, other things being equal, is proportional to the area in front view multiplied by the square of the speed. This is known as "the V^2 Law". There are, however, a number of corrections of various degrees of importance to be applied to this statement and we will now pass to a consideration of them.

Correction for Dimension Effect.—Extensive and prolonged researches carried out at the National Physical Laboratory have satisfactorily established that as long as we are considering objects of exactly the same shape (though not necessarily of the same size), and as long as the air stream meets these bodies in identical relative directions, the resistance, *i.e.* the component of the air reaction in the direction of the air stream, can be expressed accurately by the equation—

$$\text{resistance} = Kl^2V^2,$$

where l is a linear dimension of the body, V is the speed and K is a function of the product lV.*

* Variations in the viscosity of the air are neglected in this statement of the law, as they are unimportant for our purpose.

Further, it has been established that the function of lV referred to consists of the sum of a constant and terms whose importance is relatively small. If we neglect these terms entirely we have the V^2 law, which, as has been stated above, is a general approximation to the facts.

It is necessary, however, to consider more closely the quantity K. As has been stated, K is a function of lV and can therefore be plotted on lV as a base if enough experiments are available. This has been done for a great variety of objects of different kinds and the following results have been obtained :—

(1) For bodies of the high resistance class, such as round wires and cables, K may be taken as practically constant from the values of lV attained in wind tunnel tests up to the values of lV corresponding to machines in flight, so that we may use the formula—

$$\text{resistance} = (\text{a constant}) \times l^2V^2.$$

(2) For bodies of the friction class, such as well stream-lined fuselages, K decreases as lV increases over the above range to such an extent that the best approximation for the resistance is found to be the formula—

$$\text{resistance} = (\text{a constant}) \times (lV)^{1\cdot 85}.$$

(3) We may safely assume that bodies of an intermediate class, such as stream-lined struts, can be dealt with by a formula of an intermediate type.

We will now consider Case (2) in some detail. Suppose that we have to find the resistance of the hull of a flying boat which is anticipated to have minimum and maximum flying speeds of roughly 40 and 120 miles per hour respectively, and that we have to work from a wind tunnel test on a model made to a linear scale of $\frac{1}{16}$ full size, carried out at a wind speed of 27 miles per hour—conditions that are quite usual.

Let L be the value of l for the case of the actual hull, and let λ be the constant in the equation of Case (2) above, then

$$\text{resistance} = \lambda(lV)^{1\cdot 85} = \frac{\lambda l^2V^2}{(lV)^{\cdot 15}} = \frac{\lambda l^2V^2}{L^{\cdot 15}\left(\frac{lV}{L}\right)^{\cdot 15}}.$$

The values of $\left(\frac{lV}{L}\right)^{\cdot 15}$ are readily found by logarithms in given numerical cases, so that we can obtain the following table :—

Case.	$\frac{l}{L}$.	V.	$\left(\frac{lV}{L}\right)^{\cdot 15}$.	Error in applying the square law from the model test.	Error in applying the square law from the actual resistance of the full-sized body at 100 m.p.h.
Model	$\frac{1}{16}$	27	1·082	—	—
Machine at 40 m.p.h.	1	40	1·739	+ 61 per cent	− 13 per cent
Machine at 100 m.p.h.	1	100	1·995	+ 84 per cent	0
Machine at 120 m.p.h.	1	120	2·050	+ 89 per cent	+ 3 per cent

A consideration of this table leads us to adopt the following working rule for Case (2): Work out the actual full scale resistance at 100 miles per hour, and for other speeds, take the V^2 law, working from the value so obtained. This certainly introduces errors, but these are quite small for high speeds, while for low speeds, as will be evident at a later stage, resistance errors do not affect the performance of a machine at all seriously.

Passing now to the consideration of Case (3) and taking a stream-lined strut as an example, say a strut $2\frac{1}{2}$ inches wide in the direction transverse to the wind on a machine doing 120 miles per hour, we shall have to work from a wind tunnel test on a strut of similar shape but only 1 inch wide and tested at a speed of 27 miles per hour.

Then if we make the quite reasonable assumption that in this case

$$\text{resistance} = \lambda(lV)^{1\cdot 9}$$

where λ is a constant, we get

$$\text{resistance} = \frac{\lambda l^2 V^2}{L^{\cdot 1}\left(\frac{lV}{L}\right)^{\cdot 1}}$$

and we can obtain the following table:—

Case.	$\frac{l}{L}$.	V.	$\left(\frac{lV}{L}\right)^{\cdot 1}$.	Error in applying the square law from the model test.
Model	·4	27	1·269	—
Machine at 120 m.p.h. .	1	120	1·614	+ 27 per cent

In this case it is therefore usual to work on the square law direct from the model test, particularly as the error is on the safe side and only affects a moderate proportion of the total body resistance of the machine; moreover, the example analysed above is rather an extreme case.

To sum up the influence of Dimension Effect on body resistance, we may say that the square law can be used direct from model tests except for bodies coming within Case (2), namely, Fuselages, Flying Boat Hulls, Floats (if they have long fine tails, not blunt sterns), and Engine Nacelles (if they are designed on fine lines): in the case of these bodies, however, the square law is to be used from the actual full scale resistance at 100 miles per hour, which must first be found.

Correction for Angle of Attack.—Any part of the machine which is subject to air resistance is, of course, designed to have as small a resistance as practicable when it is head on to the wind. Many parts of a machine, however, are not always so placed, but are either placed permanently at an angle to the wind direction or else take up an angle to the wind when the attitude of the machine changes in flight. We will now consider these.

Fuselages, Hulls, Floats, and Similar Bodies.—The designer usually aims as far as possible at getting these placed so that they are head on to the wind when the machine is at top speed (except in the case of commercial machines, when he will of course work to the cruising speed instead), thus he makes their resistance a minimum under the conditions which are most important for practical reasons. When, however, the machine is flying more slowly and is consequently in a "tail down" attitude the resistance of such bodies as these is undoubtedly increased. This increase in resistance is accompanied by the introduction of a lifting force from the air, which tends to compensate for the loss of power due to increased resistance, at least in its influence on the rate of climb attainable. Partly because of this tendency to compensation and partly because the performance calculations

would otherwise assume an unmanageable complexity, it is customary to neglect the angle of attack effect in the case of these bodies.

Horizontal and Inclined Stream-lined Bodies such as Faired Axles and Stream-line Wires.—The same can be said of these as of those considered above: no correction for angle of attack is attempted.

Incidence Wires.—The obliquity in this case is allowed for simply by measuring their length on a front view drawing of the machine instead of taking their true length from a side view drawing.

Control Cables which are not Approximately Transverse to the Wind.—Cases of this occur in the control cables from the fuselage to the king-levers on the tail unit, in the cables from the front spar to the aileron king-levers, and in the cables from all these king-levers back on to the control surfaces themselves. Again, the lengths should be scaled from the front view drawing of the machine.

To sum up the influence of angle of attack or obliquity, it is not customary to bother about it at all except that certain wire lengths are scaled from the front view drawing. It must be admitted that this attitude is adopted not only because such obliquity has no great bearing on the performance of the machine but also because the performance calculations would become impossibly long if all these minor points were punctiliously dealt with.

Correction for Shielding.—When two similar objects are placed exactly one behind the other, the front one obviously disturbs the wind flow in such a way that the back one is subjected to a wind of less effective velocity. Therefore we might expect to find that the total resistance of the combination of two objects so placed is less than double the resistance of either considered separately. This expectation is confirmed by experiments made at the National Physical Laboratory, from which the corrections plotted in the lower curve, on page 81, have been deduced. These curves give the corrections to be applied in the case of two cables or two stream-line wires placed one behind the other, according to the fore and aft distance between centres in terms of the fore and aft dimension of either wire. These corrections are applicable to the case of double flying wires but not to the case of front and rear gap struts, since these are too widely separated for shielding to be effective.

It should be noted in this connection, however, that if one of the wires is vibrating the shielding effect will obviously disappear, or may even be replaced to some extent by something of the nature of interference. Again, the effect of angle of attack in the case of double flying wires will be to bring the back wire partially into view and so reduce shielding. For this reason the majority of designers reject this correction entirely and reckon the resistance of each of the two wires at full value, and on the whole this practice is to be recommended.

Correction for Interference.—It may be that two streamline struts are placed side by side at a small interval. This case is most likely to occur at the junction of a folding wing with the stationary centre section. It is usual to design these two struts in such a way that they form a complete stream-line shape, either by virtue of their own shape or by the shape of their fairings. When this is done the resistance of the combination can of course be obtained at once by treating it as a single streamline strut.

When, however, the designer has adopted two separate streamline struts side by side, further consideration is necessary. It appears likely on the face of it that when the struts are so close that the clear interval between them is not large in comparison to the width of either strut, the air would have less freedom of passage than when the struts are far apart, and consequently we might expect the resistance of the combination to exceed the sum of the resistances of the two struts considered separately. This expectation is confirmed by experiments made at the National Physical Laboratory, from which the upper curve of p. 81 has been prepared. This curve gives the correction to be applied for various values of the distance between the centres of the struts, in relation to the width of either. If the struts are of unequal widths, the mean of the two widths should be taken.

Numerical Values.—As has been explained above, the square law can be applied direct from the model test in the case of the blunter shapes, but it should be applied from the full scale resistance at 100 miles per hour in the case of those shapes which have really fine lines.

The best plan, therefore, is to find the full scale resistances at 100 miles per hour for *all* the objects that contribute to the total body resistance. Then by adding up we obtain a single term for the total full scale body resistance at this speed: from this we can work conveniently on the square law for all other speeds of the machine with which we are concerned.

Large Bodies.—In order to avoid the tedious process of finding the full scale resistance at 100 miles per hour from the nearest available model test in each case, formulæ based on average cases are given on page 78, and will be found to give sufficiently close approximation in practice. In working out the numerical values the speed has been taken as 100 miles per hour and a reasonable average size has been chosen.

It will be noted that cockpits and wind screens are not included in these formulæ. It is best to consider the resistances of these independently of that of the body proper. In finding the resistance of a fuselage or other body therefore, the shape to take is *the shape the body would have if no wind screens had been fitted and no cockpits cut in it.*

Cockpits.—Owing presumably to the swirling of air round the edges, a cockpit has a very high resistance. For this reason it is deemed best to estimate its resistance separately from the fuselage in which it is cut. A formula for cockpit resistance is given on page 82. In the case of tandem cockpits it is very doubtful if any effective shielding takes place, and it is therefore advisable to take all cockpits at full value for resistance. When a cockpit is fitted with a wind screen, the resistance of the latter is, of course, an addition. Cockpits which are entirely enclosed do not, of course, come under this heading, but have to be treated as modifications of the fuselage shape.

Tail Surfaces.—The rudder and fin present no difficulty as they can simply be estimated as proportional to their area. A numerical figure is given on page 78. The tail plane, however, is often so set that even at top speed the elevators are at an angle to the fixed portion of the tail plane: moreover, the relative air stream meeting the tail is seldom inclined downwards at just the angle which will give zero angle of incidence to the tail plane. For these reasons the numerical figure given on page 78 is higher than that given for the rudder and fin. Again the resistance is proportional to the area.

Struts, Stream-line Wires, and Cables.—In each case the resistance is proportional to the frontal area, which is most easily computed by measurements of length on a front view drawing of the machine coupled with reference to a schedule for the diameters. Numerical values are given on pages 80 and 82. A number of strut sections are given in order that a numerical value may be chosen which corresponds to a strut section closely approximating to the one used on the machine. King-levers and other such small parts built to stream-line strut section should

have their frontal areas measured and should be treated as struts.

Wheels.—Figures for the resistance of a wheel with different types of fairing are given on page 82, and in this case it is convenient to take the resistance as proportional to the product of the diameter and the tyre diameter.

Radiators.—When the machine is flying at top speed the radiator shutters are usually nearly closed if not quite shut. For this reason the figure given on page 83 for a radiator mounted outside the fuselage is the flat plate figure, while a compromise figure is given for the case of a nose radiator. When a retractable radiator is used, it will only have about half its area exposed at top speed, hence a lower figure (if reckoned on the total radiator area) is given for this case on page 83 .

Flat Plates.—A number of minor parts of a machine approximate to being flat plates normal to the wind: for instance, wind screens, tail skids and parts of fittings. The frontal area of these should be added up throughout the machine and multiplied by the coefficient given on page 83.

Circular Cylinders.—Under this heading are included projecting heads of engine cylinders, any unfaired wheel axles, etc. The frontal area should be added up for the whole machine and the coefficient given on page 83 used.

Miscellaneous.—Any parts of the machine subject to air resistance which remain to be dealt with must be carefully considered and estimated on the basis of their frontal area. Most designers prefer to estimate these at the "flat plate" figure, perhaps taking a somewhat reduced area in dealing with such items as look to be not very bad from the resistance point of view.

A word may be put in here about wire attachments. These are so numerous that it is best not to deal with them under this head, and for that reason the fork ends of stream-line wires and the splices and wire strainers used on cables are allowed for by adding a constant to the length of each wire, as is explained on p. 82.

Total Body Resistance.—When all the items of the body resistance have been found as described above, they must be added, and the sum is R, the total body resistance of the machine at 100 miles per hour. It is convenient, however, to note the parts of R corresponding to bodies in and not in the propeller slip stream.

Let R_1 be the part corresponding to bodies in the slip stream

and R_2 the part corresponding to bodies which are clear of it.* Let V be the velocity of the machine in miles per hour and let bV be the velocity in miles per hour added to the air by the propeller, so that the relative air velocity in the slip stream is $(1 + b)V$ miles per hour.

Then when the machine is flying under power in air of standard density—

$$\text{total body resistance} = [R_1(1 + b)^2 + R_2]\left(\frac{V}{100}\right)^2$$

while, if the machine is gliding, so that $b = 0$, we get

$$\text{total body resistance} = [R_1 + R_2]\left(\frac{V}{100}\right)^2 = R\left(\frac{V}{100}\right)^2.$$

Total Body Resistance at an Altitude.—At an altitude the density of the air is less than that at ground level. The ratio of the density of the air at an altitude to the standard air density (which differs slightly from that at ground level) is denoted by σ and the variation of σ with altitude is shown in the curve plotted on page 104.

This reduction of density has an effect on the air resistance of any body which is such that the resistance is reduced directly in the ratio of the densities.

We therefore have for a machine flying under power at an altitude—

$$\text{total body resistance} = \sigma[R_1(1 + b)^2 + R_2]\left(\frac{V}{100}\right)^2$$

and for a machine gliding at an altitude—

$$\text{total body resistance} = \sigma R\left(\frac{V}{100}\right)^2.$$

Line of Action of Body Resistance.—The height of the line of action of the body resistance is required in order to enable certain refinements to be included in the machine performance calculation if necessary. This line of action is the line of action of the resultant of the numerous parallel forces which together make up the body resistance. The ordinary method of taking moments which is used for finding the resultant of a set of parallel forces is therefore applicable.

* Bodies in front of a propeller are approximately clear of the slip stream.

The procedure is therefore as follows:—

First, note the resistances of the various items in the total body resistance,* and then the vertical distance of the line of action of each item from some convenient datum (such as the top longeron of the fuselage), using positive signs for items which are above and negative signs for items which are below the datum.

Then multiply each item of resistance by its appropriate vertical distance and form the algebraic sum of the resulting quantities.

Finally, divide this sum by the sum of the items of resistance (*i.e.* by R) and the result is the vertical distance of the required line of action above the chosen datum.

* For this calculation there is no need to deal with R_1 and R_2 separately: it is near enough to treat all the items on the same basis.

CHAPTER II.

WING CHARACTERISTICS.

General.—The properties of the wing with which we are chiefly concerned in performance calculations are k_L, the absolute lift coefficient, L/D, the ratio of the lift coefficient to the drag coefficient, and k_c, the centre of pressure coefficient.*

The original practice among designers was to note the values of these quantities at a series of angles of incidence. This practice, however, is unsatisfactory, as it does not lend itself to the application to a model test of the corrections for aspect ratio, gap/chord, stagger, wing tip shape, and dimensions, which have to be applied to the test results on a model before they can be used in calculating the performance of an actual aeroplane.

The procedure here employed is to define a quantity λ as $k_L/k_{L max}$, the ratio of the lift coefficient to the maximum value of the lift coefficient (*i.e.* to the lift coefficient at the stalling angle). A series of values of λ is then taken, ·1, ·2 . . . ·9, 1·0, and k_c, and the corrected value of L/D is noted for each, while the corrected value of $k_{L max}$ is also noted (the last, in conjunction with λ, gives k_L, of course).

This procedure enjoys the advantage of lending itself satisfactorily to the application of the necessary correcting process: the reason is that the available tests on wing tip shape, for instance, were done on a certain wing section, which wing section has a certain range of lifting angles of attack and a certain range of k_L: when, therefore, an attempt is made to estimate on an angle of attack basis, with the help of these tests, the wing tip corrections for a wing section which perhaps has a longer range of lifting angles and a higher $k_{L max}$, no result can be obtained: correspondingly, failure will result from an attempt to estimate on a k_L basis. When, however, the value of λ is taken as the basis of estimating, no such difficulties arise, since the wing and all the models made use of in the process have values of λ from 0 to 1.

* The angle of incidence comes into performance calculations only in connection with certain corrections for the action of the propeller slip stream on the wings. These will be dealt with later.

Method of Correcting.—Considering, for instance, the correction for rounded wing tips, the procedure is as follows:—tests carried out at the National Physical Laboratory on the effect of rounding off various lengths of wing tip are available. A glance at these tests shows that, broadly speaking, the rounding has a favourable effect both on the values of k_L and L/D. From the published figures it is possible by cross plotting to obtain a series of curves, one for each of a set of values of λ, giving, on a base of the span of the rounded tip divided by the chord, the value of L/D divided by that for the standard case of zero rounding (*i.e.* square tips). This has been done and the resulting curves (and a dotted curve for the $k_{L\,max}$ correction) are given on pages 92 to 95. The method of applying the correction is to take the value of $k_{L\,max}$ for the standard case and multiply it by the correction given by the dotted curve, and to take the value of L/D for the standard case at a given value of λ and multiply it by the correction given by the appropriate curve, the curves in each case being read at a point corresponding to the number of chords rounded off at each wing tip, of course. The results obtained are the values corrected for wing tip shape.

On pages 88 to 97 curves are given for finding all the necessary corrections. The various corrections are, of course, all to be multiplied together.

The weak point in the system is that corrections for, say, wing tip shape, obtained on a certain wing section, are not, strictly speaking, applicable to a different wing section. It is, however, impossible to get a more accurate prediction until all the standard corrections have been repeated on all standard wings—which would be a gigantic task.

There is another but less serious weakness in the method, namely, the assumption that the errors can be dealt with separately and their combined effect found by multiplication. This, however, is an accepted scientific principle of approximation, and there is no need to worry about it. The best defence, perhaps, of the complete procedure is that it gives close predictions of the performance of aeroplanes—which after all is all that is required.

When a test on a model of the wing to be used is available, the model corresponding to the wing in aspect ratio, gap/chord stagger and wing tip shape, and differing from it only in size and in the fact that the channel test speed is below the flying speed of the machine, it is, of course, only necessary to apply the correction for dimensions.

The Question of Correcting k_c.—The value of k_c only comes into the performance calculations of an aeroplane in determining the difference between the lift of the wing and the weight of the machine due to the vertical component of load on the tail. In other words, k_c is only used in evaluating a correction, hence corrections on k_c are really of the nature of corrections on a correction, and can therefore be neglected. It follows that we can take the values of k_c obtained on a standard test as being near enough for our purpose.

The above sufficiently explains the principles on which to obtain the wing characteristics, that is to say, the value of $k_{L max}$ corrected, the corrected values of L/D at each value of λ from ·1 to 1·0 and the uncorrected values of k_c for these values of λ.

The employment of the wing characteristics in the machine performance calculation will appear in due course.

CHAPTER III.

PROPELLER PERFORMANCE CURVES.

General.—The work of Harold Bolas* has placed in the hands of the machine designer a ready means of estimating the performance of a propeller designed to meet given conditions. The curves of pages 100 and 101 are obtained from his formulæ. They are used as indicated in Chapter X., page 102, and give the propeller performance curves, *i.e.* the plottings of P_T, the output horse-power at full *torque*, and P_R, the output power at full *revolutions*, on a base of V, the speed of the machine, for standard density air with very little trouble.

Before using Bolas' curves, however, it is necessary to determine a suitable diameter for the propeller. This is readily done with the aid of formulæ due to H. C. Watts,† which will be found in Chapter X., page 99.

It remains for us to see how a propeller, already determined for conditions of standard density air, will behave at an altitude.

Engine Power at an Altitude.—*Definitions* :—

I is the indicated horse-power.

H is the brake horse-power.

F is the frictional horse-power lost in the engine.

P_T is the effective horse-power at full torque.

P_R is the effective horse-power at full revolutions.

Q is the *indicated* torque.

N is the revolutions per minute of the engine (whereas n is used for the revolutions per minute of the propeller which are proportional to those of the engine but not necessarily equal to them).

T is the propeller thrust in pounds.

V is the machine speed in miles per hour.

All the above symbols refer to standard density air. Corresponding symbols with dashes refer to an altitude where the relative air density is σ. Corresponding symbols in italic

* See C.I.M. No. 704 issued by the Air Board.

† See "The Design of Screw Propellers for Aircraft," published by Longmans, Green & Co.

character, refer to the condition where the engine is giving both its full torque and its full revolutions in standard density air—generally called simply "the design conditions".

The Constant Torque Curve at an Altitude.—Consider a point on the constant torque curve, *i.e.* the full torque curve, in standard density air, where the effective horse-power is P_T, the velocity V, and the revolutions N, and also a point on the full torque curve at the altitude.

Let the point on the curve at the altitude be so related to the point on the curve for standard density air that

$$\frac{V'}{V} = \frac{N'}{N} \quad . \quad . \quad . \quad . \quad (1)$$

Then it follows from propeller theory that

$$\frac{H'}{H} = \frac{P_T'}{P_T} \quad . \quad . \quad . \quad . \quad (2)$$

and

$$\frac{T'}{T} = \sigma\left(\frac{V'}{V}\right)^2 \quad . \quad . \quad . \quad . \quad (3)$$

also of course

$$\frac{P_T'}{P_T} = \frac{T'V'}{TV} \quad . \quad . \quad . \quad . \quad (4)$$

Also the points we are considering are on the full torque curves,

$$\therefore Q = \mathcal{Q} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and in addition it is generally known that the full indicated torque at an altitude is σ times that for standard density air,

$$\therefore Q' = \sigma \mathcal{Q} \quad . \quad . \quad . \quad . \quad (6)$$

Also we have
$$I = H + F \quad . \quad . \quad . \quad . \quad (7)$$

and
$$I' = H' + F' \quad . \quad . \quad . \quad . \quad (8)$$

Now assuming that the frictional loss is independent of altitude and only varies as the revolutions,* we have—

$$\frac{F}{N} = \frac{\mathcal{F}}{\mathcal{N}} \quad . \quad . \quad . \quad . \quad (9)$$

and

$$\frac{F'}{N'} = \frac{\mathcal{F}}{\mathcal{N}} \quad . \quad . \quad . \quad . \quad (10)$$

also of course

$$\frac{QN}{I} = \frac{\mathcal{Q}\mathcal{N}}{\mathcal{I}} \quad . \quad . \quad . \quad . \quad (11)$$

and

$$\frac{Q'N'}{I'} = \frac{\mathcal{Q}\mathcal{N}}{\mathcal{I}} \quad . \quad . \quad . \quad . \quad (12)$$

* This assumption is justified by the fact that the rate of falling off of engine brake horse-power with altitude can be correctly predicted from it.

Again, $\sigma\left(\frac{V'}{V}\right)^3 = \sigma\left(\frac{V'}{V}\right)^2\left(\frac{V'}{V}\right)$

$= \frac{T'}{T}\frac{V'}{V}$ from equation (3)

$= \frac{P_T{}'}{P_T}$ from equation (4) . . . $\left(\frac{P_T{}'}{P_T}\right)$

$= \frac{H'}{H}$ from equation (2)

$= \frac{I' - F'}{I - F}$ from equations (7) and (8). (13)

Now $I' = \frac{Q'N'}{QN}I$ from equation (12)

$I = \frac{QN}{QN}I$ from equation (11)

$F' = \frac{N'}{N}F$ from equation (10)

and $F = \frac{N}{N}F$ from equation (9).

Using these last four equations in (13) we have—

$$\sigma\left(\frac{V'}{V}\right)^3 = \frac{\frac{Q'N'}{QN}I - \frac{N'}{N}F}{\frac{QN}{QN}I - \frac{N}{N}F}$$

$$= \frac{N'}{N}\cdot\frac{\frac{Q'}{Q}I - F}{\frac{Q}{Q}I - F}$$

$$= \frac{N'}{N}\cdot\frac{\sigma I - F}{I - F} \text{ from equations (5) and (6)}$$

$$= \frac{V'}{V}\cdot\frac{\sigma I - F}{I - F} \text{ from equation (1)}$$

$$= \sigma_1\frac{V'}{V}$$

where $\sigma_1 = \frac{\sigma - p}{q}$ . . . . . . (σ_1)

and $$p = \frac{F}{I} \quad . \quad . \quad . \quad . \quad . \quad . \quad (p)$$

and $$q = 1 - p \quad . \quad . \quad . \quad . \quad . \quad (q)$$

$$\therefore \sigma\left(\frac{V'}{V}\right)^2 = \sigma_1$$

$$\therefore V' = V\sqrt{\frac{\sigma_1}{\sigma}} \quad . \quad . \quad . \quad . \quad . \quad (V')$$

Again, $$\frac{P_T'}{P_T} = \sigma\left(\frac{V'}{V}\right)^3 \quad \text{from equation } \left(\frac{P_T'}{P_T}\right)$$

$$= \sigma\left(\frac{\sigma_1}{\sigma}\right)^{\frac{3}{2}}$$

$$= \sigma_1\sqrt{\frac{\sigma_1}{\sigma}}$$

$$\therefore P_T' = P_T\sigma_1\sqrt{\frac{\sigma_1}{\sigma}} \quad . \quad . \quad . \quad . \quad . \quad (P_T')$$

Equations (V') and (P_T') enable us to plot the full torque propeller curve at the altitude from the one for standard density air, with the aid of equations (σ_1), (p), and (q). The full torque at the altitude, corresponding to full throttle at the altitude, *is σ times the full torque for standard density air.*

As for p and q, we may take $p = \cdot 262$ and $q = \cdot 738$ for rotary engines, and $p = \cdot 161$ and $q = \cdot 839$ for stationary engines.

The Constant Revolutions Curve at an Altitude :—

On this curve for P_R' on a base V', the revolutions are *the full revolutions for standard density air* (unlike the case of the curve for P_T' on V', where the torque is reduced at the altitude).

Consider a point on the P_R, V curve and one on the P_R', V' curve, and let them be related by the equation—

$$\frac{V'}{V} = \frac{N'}{N}.$$

Then as before we have—

$$\frac{T'}{T} = \sigma\left(\frac{V'}{V}\right)^2$$

and $$\frac{P_R'}{P_R} = \frac{T'V'}{TV}$$

$$\therefore \frac{P_R'}{P_R} = \sigma\left(\frac{V'}{V}\right)^3$$

but N′ and N are equal, since they are each equal to N,

$$\therefore V' = V \quad . \quad . \quad . \quad . \quad (V')$$

$$\therefore P_R' = \sigma P_R \quad . \quad . \quad . \quad . \quad (P_R')$$

Equations (V′) and (P_R') suffice to determine the P_R', V′ curve for any altitude from the P_R, V curve.

Note that values of V′ determined in this work will differ from those determined in the previous investigation of P_T'.

It will perhaps make for clarity of thought to remember that at an altitude, if the throttle is left full open, the P_T' curve will be developed. The range of this curve, however, at speeds higher than its intersection with the P_R' curve, involve revolutions greater than N: the engine should therefore be throttled at these speeds till the performance falls on to the P_R' curve in this region.

The P_R' curve between the origin and the intersection speed can never be developed: it is used, however, in later work connected with throttling, and that is the reason why we retain it.

CHAPTER IV.

MACHINE PERFORMANCE CURVE.

General.—The machine performance curve is a curve giving P, the effective horse-power required by the machine in order that horizontal flight may be maintained, plotted on V, the speed of flight in miles per hour as a base.

In the first place, this curve is worked out for standard density air, but it can also be found for any given altitude, in which case it is a plotting of P', the effective horse-power required for horizontal flight at that altitude, on a base of V', the flying speed at that altitude.

The machine performance curve can be applied directly to problems of top speed, landing speed, cruising economy, length of run to get off the ground, and in dealing with the rate of climb of the machine.

In determining the machine performance curve the procedure consists essentially in first determining the speed for a given value of the lift coefficient and then obtaining P by the relation between P and the total resistance and velocity.

There are, however, certain corrections to be taken account of which together make the problem one of a somewhat formidable complexity. Fortunately it is possible to get over the difficulty by splitting up the problem into a series of methods of successive degrees of accuracy. The methods will be given separately hereunder, as it is not always necessary to use the more complex methods, and in that case easier ones can be used.

The machine performance curve will now be worked out to various degrees of approximation, all the cases usually used being dealt with separately.

First Method.—*Assumptions.*—Line of flight horizontal. Propeller thrust horizontal at all speeds of flight and passing through the centre of head resistance. Tail air load neglected. Slip stream neglected.

Definitions: P and V have already been defined above.

W is the total weight of the machine in pounds.

T is the total resistance of the machine in pounds.

L is the lift of the wings in pounds.
S is the wing area in square feet.
R has been defined in Chapter I.
L/D, k_L, k_{Lmax}, and λ have been defined in Chapter II.
Now on our assumptions—

$$P = \frac{VT}{375} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$T = R\left(\frac{V}{100}\right)^2 + \frac{W}{L/D} \quad . \quad . \quad . \quad . \quad (2)$$

$$L = \cdot 00237 k_L S(1 \cdot 467V)^2 \quad . \quad . \quad . \quad (3)$$

$$L = W \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$k_L = \lambda k_{Lmax} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

From equations (3), (4), and (5)—

$$V^2 = \frac{1}{\cdot 00237(1 \cdot 467)^2} \frac{L}{k_L S}$$

$$= \frac{1}{\cdot 00237(1 \cdot 467)^2} \frac{W}{\lambda k_{Lmax} S}$$

$$\therefore V = \sqrt{\frac{a}{\lambda}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (V)$$

where

$$a = \frac{1}{\cdot 00237(1 \cdot 467)^2} \frac{W}{S k_{Lmax}} = \frac{196W}{S k_{Lmax}} \quad . \quad (a)$$

$$\therefore R\left(\frac{V}{100}\right)^2 = \frac{Ra}{10{,}000\lambda}$$

$\therefore$ from equation (2)—

$$T = \frac{\beta}{\lambda} + \frac{W}{L/D} \quad . \quad . \quad . \quad . \quad . \quad (T)$$

where

$$\beta = \frac{Ra}{10^4} \quad . \quad . \quad . \quad . \quad . \quad . \quad (\beta)$$

Equations (1), (V), (a), (T), and (β) are in such a form that a tabular method can be conveniently applied to them (see Chapter XI.).

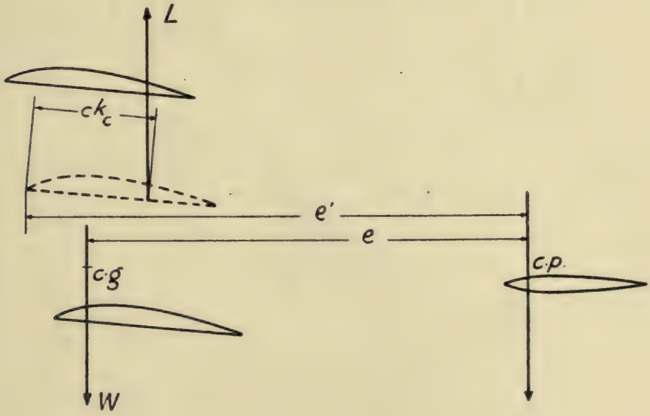
The values of L/D and k_{Lmax} used above are of course the corrected values taken from the wing characteristic.

The above equations having been solved tabularly, we have values of V and P for values of λ from ·1 to 1·0: these values of V and P are the data required for plotting the performance curve for standard density air under the assumptions of the First Method.

Second Method.—*Assumptions.*—Line of flight horizontal. Propeller thrust horizontal at all speeds of flight and passing through the centre of head resistance. Slip stream neglected.

Definitions.—These are the same as in the First Method, but in addition:—

l is the horizontal distance in feet from the c.g. of the machine to the c.p. of the tail plane.*



l' is the horizontal distance in feet from the leading edge of the equivalent chord to the c.p. of the tail plane.

c is the chord of the wing in feet.

k_c has been defined in Chapter II.

The equivalent chord is of course the chord of the equivalent plane, which is an imaginary monoplane situated between the top and bottom planes of the actual biplane, but ·55 of the gap from the lower plane: it has the property that for purposes of moments it can be considered to replace the actual biplane. Hence the line of action of L cuts the equivalent chord at a distance ck_c from the leading edge (see figure).

We now have on our present assumptions a set of equations very like those of the First Method, namely—

$$P = \frac{VT}{375} \quad . \quad . \quad . \quad . \quad (1)$$

* It is sufficiently accurate for our purpose to assume that the c.p. of the tail plane is at the front tail plane spar.

$$T = R\left(\frac{V}{100}\right)^2 + \frac{L}{L/D} \quad . \quad . \quad . \quad (2)$$

$$L = \cdot 00237 k_L S(1 \cdot 467 V)^2 \quad . \quad . \quad . \quad (3)$$

$$Wl = L(l' - ck_c) \quad . \quad . \quad . \quad . \quad (4)$$

$$k_L = \lambda k_{L max} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

equation (4) being obtained by taking moments about the c.p. of the tail, neglecting the cosines of small obliquities (see figure). From equation (4) we obtain—

$$L/W = \frac{l}{l' - ck_c} = \frac{\frac{l}{c}}{\frac{l'}{c} - k_c} = \frac{\gamma}{\delta - k_c} \quad . \quad (L/W)$$

where $$\gamma = \frac{l}{c} \quad . \quad . \quad . \quad . \quad (\gamma)$$

and $$\delta = \frac{l'}{c} \quad . \quad . \quad . \quad . \quad (\delta)$$

From equations (3), (5), and (L/W) we have—

$$V^2 = \frac{1}{\cdot 00237(1 \cdot 467)^2} \frac{L}{k_L S}$$

$$= \frac{1}{\cdot 00237(1 \cdot 467)^2} \frac{WL/W}{\lambda k_{L max} S}$$

$$\therefore V = \sqrt{\frac{aL/W}{\lambda}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (V)$$

where $$a = \frac{1}{\cdot 00237(1 \cdot 467)^2} \frac{W}{Sk_{L max}} = \frac{196W}{Sk_{L max}} \quad . \quad . \quad (a)$$

$$\therefore R\left(\frac{V}{100}\right)^2 = \frac{RaL/W}{10{,}000\lambda}$$

$\therefore$ from equation (2)—

$$T = \frac{\beta L/W}{\lambda} + \frac{WL/W}{L/D} \quad . \quad . \quad . \quad . \quad (T)$$

where $$\beta = \frac{Ra}{10^4} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\beta)$$

Equations (1), (L/W), (γ), (δ), (V), (a), (T), and (β) are in such a form that a tabular method can be conveniently applied to them (see Chapter XI.).

The values of L/D and $k_{L max}$ used above are, of course, the corrected values taken from the wing characteristic.

The above equations having been solved tabularly, we have values of V and P for values of λ from ·1 to 1·0: these values of V and P are the data required for plotting the performance curve for standard density air under the assumptions of the Second Method.

Third Method.—*Assumptions.*—Line of flight horizontal. Propeller thrust horizontal at all speeds of flight and passing through the centre of head resistance.

Definitions.—These are the same as in the Second Method, but in addition—

R_1 and R_2 have the definitions given in Chapter I.

S, as before, is the *total* wing area in square feet.

S' is the area of wing affected by the slip stream.

d is the propeller diameter in inches.

$(1 + b)V$ is the total slip stream velocity in miles per hour.

Seeing that after all the slip stream effect is only a correction, it follows that corrections on it will not have much influence on the main problem. It is therefore sufficiently accurate to take the parts of the machine which fall inside the propeller circle in front view as being subject to slip stream action, and the rest as being clear of it. This applies also to the wings: the total length of top and bottom leading edge falling within the propeller disc in front view is first measured, and then S' is estimated as S multiplied by this length and divided by the total length of top and bottom leading edges for the whole machine.

We now have, somewhat as before—

$$P = \frac{VT}{375} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$T = [(1 + b)^2 R_1 + R_2]\left(\frac{V}{100}\right)^2 + \frac{L}{L/D} \quad . \quad . \quad (2)$$

$$L = \cdot 00237 k_L [S - S' + (1 + b)^2 S'](1 \cdot 467V)^2 \quad . \quad (3)$$

$$Wl = L(l' - ck_c) \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$k_L = \lambda k_{L max} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$T = \frac{\cdot 00237}{2} \frac{\pi}{4} \left(\frac{d}{12}\right)^2 (1 \cdot 467V)^2 b(b + 2) \quad . \quad . \quad (6)$$

Equation (6) is taken from equation (8), Chapter VI. of "The Design of Screw Propellers for Aircraft," by H. C. Watts.*

From equation (4) we obtain, as in the Second Method—

* Published by Messrs. Longmans, Green & Co.

$$\text{L/W} = \frac{\gamma}{\delta - k_c} \quad . \quad . \quad . \quad (\text{L/W})$$

where
$$\gamma = \frac{l}{c} \quad . \quad . \quad . \quad . \quad . \quad (\gamma)$$

and
$$\delta = \frac{l'}{c} \quad . \quad . \quad . \quad . \quad . \quad (\delta)$$

Equation (6) may be written—

$$\text{T} = \frac{1\cdot39}{10^5}d^2\text{V}^2B \quad . \quad . \quad . \quad . \quad (6')$$

where
$$B = (1 + b)^2 - 1.$$

Also equations (2) and (3) can be replaced by—

$$\text{T} = (B\text{R}_1 + \text{R}_1 + \text{R}_2)\left(\frac{\text{V}}{100}\right)^2 + \frac{\text{L}}{\text{L/D}} \quad . \quad . \quad (2')$$

and
$$\text{L} = \cdot0051k_{\text{L}}(\text{S} + B\text{S}')\text{V}^2 \quad . \quad . \quad . \quad . \quad (3')$$

By rearranging these three equations we have—

$$B = \frac{10^5}{1\cdot39d^2}\frac{\text{T}}{\text{V}^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6'')$$

$$\frac{\text{T}}{\text{V}^2} = \frac{\text{R}_1}{10^4}B + \frac{\text{R}_1 + \text{R}_2}{10^4} + \frac{\text{L/V}^2}{\text{L/D}} \quad . \quad . \quad . \quad (2'')$$

$$\text{L/V}^2 = \cdot0051k_{\text{L}}\text{S} + \cdot0051k_{\text{L}}\text{S}'B \quad . \quad . \quad . \quad (3'')$$

Substitute for B from equation (6″) in equations (2″) and (3″) and we obtain—

$$\frac{\text{T}}{\text{V}^2} = \frac{10\text{R}_1}{1\cdot39d^2}\frac{\text{T}}{\text{V}^2} + \frac{\text{R}_1 + \text{R}_2}{10^4} + \frac{\text{L/V}^2}{\text{L/D}}$$

and
$$\text{L/V}^2 = \cdot0051k_{\text{L}}\text{S} + \frac{510k_{\text{L}}\text{S}'}{1\cdot39d^2}\frac{\text{T}}{\text{V}^2}.$$

These two equations can be written—

$$a'\frac{\text{T}}{\text{V}^2} = b' + \frac{\text{L/V}^2}{\text{L/D}}$$

and
$$c'\frac{\text{T}}{\text{V}^2} = \frac{\text{L/V}^2}{\lambda} - d'$$

where
$$a' = 1 - \frac{7\cdot2\text{R}_1}{d^2} \quad . \quad . \quad . \quad (a')$$

$$b' = \frac{\text{R}_1 + \text{R}_2}{10^4} \quad . \quad . \quad . \quad . \quad (b')$$

$$c' = \frac{367 k_{L\,max} S'}{d^2} \quad . \quad . \quad . \quad . \quad (c')$$

and
$$d' = \cdot 0051 k_{L\,max} S \quad . \quad . \quad . \quad (d')$$

From these two equations we obtain—

$$b'c' + c'\frac{L/V^2}{L/D} = a'c'\frac{T}{V^2} = \frac{a'L/V^2}{\lambda} - a'd'$$

$$\therefore \ L/V^2\left(\frac{a'}{\lambda} - \frac{c'}{L/D}\right) = b'c' + a'd'$$

$$\therefore \ V = \sqrt{\frac{\dfrac{a'}{\lambda} - \dfrac{c'}{L/D}}{a'd' + b'c'} W \ L/W} \quad . \quad . \quad . \quad (V)$$

Also from the same two equations we obtain—

$$a'L/D\frac{T}{V^2} - b'L/D = L/V^2 = c'\lambda\frac{T}{V^2} + d'\lambda$$

$$\therefore \ \frac{T}{V^2}(a'L/D - c'\lambda) = b'L/D + d'\lambda$$

$$\therefore \ \frac{T}{V^2} = \frac{b'L/D + d'\lambda}{a'L/D - c'\lambda} = \frac{\dfrac{b'}{\lambda} + \dfrac{d'}{L/D}}{\dfrac{a'}{\lambda} - \dfrac{c'}{L/D}}$$

$\therefore$ from equation (1)—

$$P = \frac{\dfrac{b'}{\lambda} + \dfrac{d'}{L/D}}{\dfrac{a'}{\lambda} - \dfrac{c'}{L/D}} \frac{V^3}{375} \quad . \quad . \quad . \quad (P)$$

Equations (L/W), (γ), (δ), (a'), (b'), (c'), (d'), (V), and (P) are in such a form that a tabular method can be applied to them, though it does not work out so simply as in the cases of the First and Second Methods (see Chapter XI.).

The values of L/D and $k_{L\,max}$ used above are, of course, the corrected values taken from the wing characteristic.

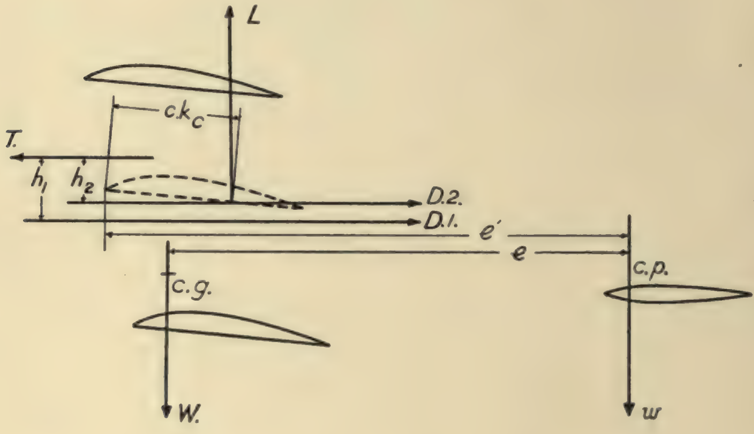
The above equations having been solved tabularly, we have values of V and P for values of λ from $\cdot 1$ to $1 \cdot 0$: these values of V and P are the data required for plotting the performance curve for standard density air under the assumptions of the Third Method. The extension of this method to the case of multi-propeller machines presents no special difficulty.

Fourth Method.—*Assumptions.*—Line of flight horizontal. Propeller thrust horizontal at all speeds of flight.

Definitions.—These are the same as in the Third Method, but in addition—

h_1 is the height in feet of the centre line of the propeller shaft above the line of action of the body resistance.

h_2 is the height in feet of the centre line of the propeller shaft above a point half way between the top and bottom leading edges: this point can be taken as being on the line of action of the wing resistance.



w is the air download on the tail plane in pounds.

D_1 is the body resistance in pounds.

D_2 is the wing resistance in pounds.

We now have somewhat as before—

$$P = \frac{VT}{375} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$T = [(1 + b)^2R_1 + R_2]\left(\frac{V}{100}\right)^2 + \frac{L}{L/D} \quad . \quad . \quad . \quad (2)$$

$$L = \cdot 00237k_L[S - S' + (1 + b)^2S'](1 \cdot 467V)^2 \quad . \quad (3)$$

Now take moments about the point on the line of action of the propeller thrust which is vertically above the centre of gravity (see figure) and we get—

$$L(ck_c - l' + l) + D_1h_1 + D_2h_2 = wl.$$

Also we have of course—

$$w = L - W$$

$$D_1 = T - D_2$$

and
$$D_2 = \frac{L}{L/D}$$

$$\therefore L(ck_c - l' + l) + Th_1 - D_2h_1 + D_2h_2 = (L - W)l$$

$$\therefore L(ck_c - l' + l) + Th_1 - \frac{L}{L/D}(h_1 - h_2) = (L - W)l$$

$$\therefore Wl + Th_1 = L\left[l' - ck_c + \frac{h_1 - h_2}{L/D}\right] \quad . \quad . \quad (4)$$

And again as before—

$$k_L = \lambda k_{L max} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and
$$T = \frac{\cdot 00237}{2}\frac{\pi}{4}\left(\frac{d}{12}\right)^2(1\cdot 467V)^2 b(b + 2) \quad . \quad . \quad (6)$$

Now as in the Third Method, equations (2), (3), and (6) can be replaced by—

$$\frac{T}{V^2} = \frac{R_1}{10^4}B + \frac{R_1 + R_2}{10^4} + \frac{L/V^2}{L/D}$$

$$L/V^2 = \cdot 0051k_L S + \cdot 0051k_L S'B$$

and
$$B = \frac{10^5}{1\cdot 39d^2}\frac{T}{V^2}$$

where
$$B = (1 + b)^2 - 1.$$

Again, as the Third Method, B can be eliminated, leading to the two equations—

$$a'\frac{T}{V^2} = b' + \frac{L/V^2}{L/D}$$

and
$$c'\frac{T}{V^2} = \frac{L/V^2}{\lambda} - d'$$

where
$$a' = 1 - \frac{7\cdot 2R_1}{d^2} \quad . \quad . \quad . \quad (a')$$

$$b' = \frac{R_1 + R_2}{10^4} \quad . \quad . \quad . \quad . \quad (b')$$

$$c' = \frac{367k_{L max}S'}{d^2} \quad . \quad . \quad . \quad (c')$$

and
$$d' = \cdot 0051k_{L max}S \quad . \quad . \quad . \quad (d')$$

Further, as in the Third Method, these two equations lead to—

$$\frac{L}{V^2} = \frac{b'c' + a'd'}{\frac{a'}{\lambda} - \frac{c'}{L/D}}$$

and
$$\frac{T}{V^2} = \frac{\frac{b'}{\lambda} + \frac{d'}{L/D}}{\frac{a'}{\lambda} - \frac{c'}{L/D}}.$$

Now put
$$\frac{a'}{\lambda} - \frac{c'}{L/D} = \theta \quad . \quad . \quad . \quad . \quad (\theta)$$

$$\frac{b'}{\lambda} + \frac{d'}{L/D} = \phi \quad . \quad . \quad . \quad . \quad (\phi)$$

and
$$a'd' + b'c' = \psi \quad . \quad . \quad . \quad . \quad (\psi)$$

$$\therefore \frac{L}{V^2} = \frac{\psi}{\theta}$$

and
$$\frac{T}{V^2} = \frac{\phi}{\theta}.$$

Substitute these values of $\frac{L}{V^2}$ and $\frac{T}{V^2}$ in equation (4) and we obtain—

$$\frac{Wl}{V^2} = -\frac{\phi h_1}{\theta} + \frac{\psi}{\theta}\left[l' - ck_c + \frac{h_1 - h_2}{L/D}\right]$$

$$\therefore V = \sqrt{\frac{Wl\theta}{\psi\left[l' - ck_c + \frac{h_1 - h_2}{L/D}\right] - \phi h_1}} \quad . \quad . \quad (V)$$

$\therefore$ from equation (1)—

$$P = \frac{\phi V^3}{375\theta} \quad . \quad . \quad . \quad . \quad (P)$$

Equations (V) and (P), with the aid of equations (θ), (ϕ), (ψ), (a'), (b'), (c'), and (d') are in such a form that a tabular method can be applied to them (see Chapter XI.).

The values of L/D and $k_{L\,max}$ used above are of course the corrected values taken from the wing characteristic.

The above equations having been solved tabularly, we have values of V and P for values of λ from ·1 to 1·0: these values of V and P are the data required for plotting the performance curve for standard density air under the assumptions of the Fourth

Method. The extension of this method to the case of multi-propeller machines presents no special difficulty.

Comparison of the Above Four Methods.—Examples (1) to (4) inclusive in Chapter XVIII., pages 155 to 161 inclusive, show the different numerical results obtained *on the same machine* by following the four different methods of calculation.

A comparison of the four curves on page 157 shows that there is little difference between the First Method and the Second Method: this being so it is advisable not to use the Second Method in ordinary work, as the slightly quicker First Method gives practically identical results. It must be remembered, however, that if the centre of gravity of the machine is unusually placed, the difference between the methods is greater: this applies particularly to flying boats having a negative tail in the slip stream and the c.g. far forward as a means of preventing a sudden stall on engine failure.

Comparing now the Third Method with the Fourth Method, we again find little to choose, so that it is usually preferable to use the quicker Third Method: this, however, again may not be advisable in the case of flying boats, since the unusually high centre of propeller thrust which is inseparable from this type causes the difference between the two methods to be greater than in the examples under consideration.

For ordinary work, therefore, the choice lies between the First Method and the Third Method. Here we see a wide departure, so that the selection of the approximate First Method instead of the practically accurate Third Method, in the case under consideration, though it would not cause much error on the climb, would result in the top speed being over-estimated by about 2 miles per hour and in the landing speed under power being over-estimated by about 4 miles per hour—the landing speed with the engine shut off (which is usually taken as the specified landing speed) must, of course, be taken from curve (1 or 2) in any case.

This being so it is advisable to use the First Method always when doing rough or first approximation work, the Second Method practically never, the Third Method always for accurate work except on flying boats, and the Fourth Method always for accurate work on flying boats.

Inclination of the Propeller Shaft. — The assumption hitherto made that the propeller shaft is horizontal at all speeds of flight is not of course true. Consequently it is not really correct to assume, as has been done, that there is no vertical

component of thrust, and that the propeller slip stream meets the wings at the same angle as the undisturbed air would do.

Unfortunately, the approximation cannot be carried any farther than has been done in the Third Method, if a complete and easily workable solution is desired.

The outstanding error due to this cause is not serious unless the propeller shaft is mounted in the machine at an unusual angle to the chord of the wing. This, however, is often done in machines designed to fly off the decks of ships, in order to obtain the advantage which can be got in this way.

The whole question of the effect on the performance of an inclined propeller shaft is therefore postponed to Chapter VI.

Non-Horizontal Flight.—The consideration of the problem when the line of flight is not horizontal belongs to the question of the rate of climb of the machine: it is therefore appropriate to postpone it to Chapter V., page 42.

Flight at an Altitude.—For the purposes of aeroplane performance calculations flight at an altitude is distinguished from flight in standard density air solely by the fact that σ, the ratio of the air density at the altitude to the standard air density, is not unity. We will now consider the influence of σ on the calculations of the machine performance curve.

As an example of the conversion process we will follow the proof of the Fourth Method, page 28.

The dimensions and weight of the machine will be the same at the altitude, also the values of λ, L/D and $k_{L\,max}$: hence also the quantities a', b', c', d', θ, ϕ, and ψ will be the same at the altitude, remembering that R_1 and R_2 are not the resistances at the altitude, but in standard density air.

Let V, P, T, L, w, D_1, D_2, and b refer to standard density air, but let the corresponding quantities at the altitude be V′, P′, T′, L′, w', D_1', D_2', and b'.

Then, analogously to the proof of the Fourth Method we shall have, for flight at the altitude, the equations—

$$P' = \frac{V'T'}{375} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$T' = \sigma[(1 + b')^2R_1 + R_2]\left(\frac{V'}{100}\right)^2 + \frac{L'}{L/D} \quad . \quad . \quad . \quad (2)$$

$$L' = \sigma \times \cdot 00237k_L[S - S' + (1 + b')^2S'](1\cdot 467V')^2 . \quad (3)$$

also
$$L'(ck_c - l' + l) + D_1'h_1 + D_2'h_2 = w'l$$
$$w' = L' - W$$

$$D_1' = T' - D_2'$$

and
$$D_2' = \frac{L'}{L/D}$$

leading to
$$Wl + T'h_1 = L'\left[l' - ck_c + \frac{h_1 - h_2}{L/D}\right] \quad . \quad . \quad (4)$$

$$k_L = \lambda k_{L max} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$T' = \frac{\sigma \times \cdot 00237}{2} \frac{\pi}{4}\left(\frac{d}{12}\right)^2 (1 \cdot 467V')^2 b'(b' + 2) \quad . \quad (6)$$

Now let $P_1' = \frac{P'}{\sigma}$, $T_1' = \frac{T'}{\sigma}$, $L_1' = \frac{L'}{\sigma}$, and $W_1 = \frac{W}{\sigma}$.

Then the above six equations become the following :—

$$P_1' = \frac{V'T_1'}{375} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1')$$

$$T_1' = [(1 + b')^2R_1 + R_2]\left(\frac{V'}{100}\right)^2 + \frac{L_1'}{L/D} \quad . \quad . \quad (2')$$

$$L_1' = \cdot 00237k_L[S - S' + (1 + b')^2S'](1 \cdot 467V')^2 \quad . \quad (3')$$

$$W_1l + T_1'h_1 = L_1'\left[l' - ck_c + \frac{h_1 - h_2}{L/D}\right] \quad . \quad . \quad . \quad (4')$$

$$k_L = \lambda k_{L max} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5')$$

$$T_1' = \frac{\cdot 00237}{2} \frac{\pi}{4}\left(\frac{d}{12}\right)^2 (1 \cdot 467V')^2 b'(b' + 2) \quad . \quad (6')$$

Now comparing these six equations with the six equations of the Fourth Method, we see that they are the same except that P_1', V', T_1', b', L_1', and W_1 have been written in place of P, V, T, b, L, and W. Therefore the solution of our present six equations can be got by making this substitution in equations (V) and (P) of the Fourth Method, page 30. We therefore have for flight at an altitude—

$$V' = \sqrt{\frac{W_1l\theta}{\psi\left[l' - ck_c + \frac{h_1 - h_2}{L/D}\right] - \phi h_1}}$$

$$P_1' = \frac{\phi V'^3}{375\theta},$$

remembering that θ, ϕ, and ψ have the same values as before

Comparing these two equations with equations (V) and (P) of page 30 we have—

$$V' = \sqrt{\frac{W_1}{W}}\, V = \frac{1}{\sqrt{\sigma}} V \qquad . \quad . \quad . \quad . \quad (V')$$

and
$$P_1' = \left(\frac{V'}{V}\right)^3 P$$

$$\therefore P' = \sigma P_1' = \sigma\left(\frac{V'}{V}\right)^3 P = \sigma\left(\frac{1}{\sqrt{\sigma}}\right)^3 P = \frac{1}{\sqrt{\sigma}} P \qquad . \quad (P')$$

Hence we have the rule that to obtain the machine performance curve at an altitude, we find the value of σ from the curve on page 104, and then divide the values of the speed and horse-power in standard density air (*i.e.* V and P) by $\sqrt{\sigma}$ to get the values of the speed and horse-power at the altitude (*i.e.* V' and P').

The same rule would have resulted if we had followed the proof of either of the other three methods, but there is no need to go into that fully, as the fact is obvious.

CHAPTER V.

AIR PERFORMANCE.

General.—By the air performance of a machine is meant the numerical evaluation of everything that the machine can do when air borne. The subject can be conveniently divided into three parts according to the proportion of the available engine power which is being used. We shall therefore consider the subject of air performance under three main heads, Gliding, Full Power, and Throttled.

I. GLIDING FLIGHT.

General.—Referring to the four methods of machine performance calculation of Chapter IV., we observe that the Third Method and the Fourth Method are not applicable, seeing that we are supposing that the engine has broken down or else has been intentionally switched off, so that there is no propeller slip stream.

Of the two remaining methods, the Second Method is the accurate one to use, but the First is generally close enough to the mark except in the case of a flying boat. In what follows therefore under the present heading of Gliding Flight, when the machine performance curve is referred to it is to be understood that it is to have been found either by the First Method or the Second Method.

Landing Speed on Glide.—When making a landing without his engine the pilot, of course, pulls his control back at the last moment so that when it actually lands the machine is flying horizontally. In this case, therefore, the Second Method may be taken as accurate (or the First Method as a close approximation) without further investigation.

For the purpose of finding the landing speed, however, we do not need the whole performance curve but only the one point on it corresponding to $\lambda = 1 \cdot 0$.

First Method.—Referring to page 22 we find that under the assumptions of the First Method of Chapter IV.—

$$V = \sqrt{\frac{a}{\lambda}}$$

where $$a = \frac{196W}{Sk_{L\,max}}$$

$\therefore$ for our case where $\lambda = 1{\cdot}0$ we have that the landing speed on glide is given by the equation—

$$V = \sqrt{\frac{196W}{Sk_{L\,max}}}.$$

Now referring to page 34 we see that the corresponding formula at an altitude is—

$$V' = \frac{V}{\sqrt{\sigma}} = \sqrt{\frac{196W}{\sigma Sk_{L\,max}}}$$

though whether there is any useful meaning in the conception of a "gliding landing speed at an altitude" is doubtful.

Second Method.—Following the Second Method of Chapter IV., we have, from page 24—

$$V = \sqrt{\frac{aL/W}{\lambda}}$$

where $$a = \frac{196W}{Sk_{L\,max}}$$

$$L/W = \frac{\gamma}{\delta - k_c}$$

$$\gamma = \frac{l}{c}$$

$$\delta = \frac{l'}{c}$$

and, in our case, $\lambda = 1{\cdot}0$.

$\therefore$ the landing speed on glide is given by the equation—

$$V = \sqrt{\frac{196Wl}{Sk_{L\,max}(l' - ck_c)}}$$

where k_c is the centre of pressure coefficient corresponding to $\lambda = 1{\cdot}0$, *i.e.* to the stalling angle.

In most machines there is a small difference between this value of V and that calculated above: the difference is not great, but if accuracy is required, this formula is the one to use.

Again, for what it is worth, we have the "gliding landing speed at an altitude" as given by the equation—

$$V' = \sqrt{\frac{196Wl}{\sigma S k_{L max}(l' - ck_c)}}.$$

Gliding Angle.—A comparison of curves (1) and (2) on page 157 will show that it is only at speeds close to the stalling speed that the First Method and the Second Method give appreciably different results. We will therefore content ourselves with the simple assumptions of the First Method (suitably modified for descending flight) in investigating the gliding angle.

The forces acting on the machine are therefore only its total weight W, the air lift L at right angles to the descending path of steady flight, and the total air resistance of wings and body T acting along the path of flight.

Let θ be the angle made by the flight path with the horizon: then θ is the gliding angle in still air.

We will use similar notation to that of Chapter IV., and will consider the machine to be gliding at an altitude, then by resolving horizontally we have—

$$L \sin \theta = T \cos \theta \quad . \quad . \quad . \quad . \quad (1)$$

also, of course, we have—

$$T = \sigma R\left(\frac{V'}{100}\right)^2 + \frac{L}{L/D} \quad . \quad . \quad . \quad . \quad (2)$$

$$L = \sigma \times \cdot 00237 k_L S(1 \cdot 467 V')^2 \quad . \quad . \quad . \quad (3)$$

and

$$k_L = \lambda k_{L max} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

From equations (3) and (4) we have—

$$L = \cdot 0051 \sigma \lambda k_{L max} S V'^2 \quad . \quad . \quad . \quad . \quad (L)$$

Substituting for L from this equation in equation (2), we have—

$$T = \frac{\sigma R V'^2}{10^4} + \cdot 0051 \frac{\sigma \lambda k_{L max} S V'^2}{L/D} \quad . \quad . \quad (T)$$

From equations (1), (L), and (T) we have—

$$\tan \theta = \frac{T}{L} = \frac{\frac{\sigma R V'^2}{10^4} + \cdot 0051 \frac{\sigma \lambda k_{L max} S V'^2}{L/D}}{\cdot 0051 \sigma \lambda k_{L max} S V'^2}$$

$$= \frac{R}{51 \lambda k_{L max} S} + \frac{1}{L/D}$$

$$\therefore \tan \theta = \frac{a}{\lambda} + \frac{1}{L/D} \quad . \quad . \quad . \quad . \quad . \quad (\tan \theta)$$

where

$$a = \frac{R}{51 k_{L max} S} \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

It will be noticed that σ has dropped out: from this we deduce that the gliding angle in still air for a given value of λ is independent of altitude.

Equations ($\tan\theta$) and (a) are in a convenient form for tabular treatment for values of λ from ·1 to 1·0 (see Chapter XII., page 117).

Such a table can be drawn up and then, by plotting, the minimum value of $\tan\theta$ can be found: this is the tangent of the "best gliding angle in still air"—often referred to as the tangent of the "gliding angle," simply.

Gliding in a Wind.—If there is a steady horizontal head wind* of v' miles per hour at the altitude in question, it will be clear that a glide will probably reach further if the above "best gliding angle in still air" is departed from in the direction of faster speed relative to the air. For this reason the case of gliding in a wind has to be treated separately from the case of gliding in still air.

We shall consider the problem at a definite altitude, and shall employ the same notation as in the previous section, remembering that θ and V' are now *relative to the air*. Let ϕ be the angle of glide relative to the ground.

As before, we have—

$$\tan\theta = \frac{a}{\lambda} + \frac{1}{L/D} \quad . \quad . \quad . \quad . \quad . \quad (\tan\theta)$$

$$a = \frac{R}{51 k_{L max} S} \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

$$L = \cdot 0051 \sigma \lambda k_{L max} S V'^2 \quad . \quad . \quad . \quad . \quad (L)$$

and

$$T = \frac{\sigma R V'^2}{10^4} + \cdot 0051 \frac{\sigma \lambda k_{L max} S V'^2}{L/D} \quad . \quad . \quad . \quad (T)$$

and in addition, by resolving vertically—

$$W = L \cos\theta + T \sin\theta \quad . \quad . \quad (W)$$

The last equation can be written—

$$W = \cos\theta [L + T \tan\theta]$$

* Winds are seldom steady in magnitude or direction, are often not horizontal, and generally vary considerably both in magnitude and direction with altitude. Consequently the investigation which follows, like other later investigations into which a wind enters, has an appearance of getting down to hard facts which is largely spurious.

The results of such investigations, therefore, though a better guide to the pilot navigator than they would be if wind was neglected, are only a guide after all.

For information on the variability of wind with altitude, see "Manual of Meteorology," Part IV., by Sir Napier Shaw, published by The Cambridge University Press.

Now in practice the angle θ will be round about $7°$ or less, so that $\cos\theta$ will be about ·993, or even nearer to unity.

Therefore we do not lose much in accuracy by dropping $\cos\theta$ from the above equation, leading to—

$$W = L + T\tan\theta.$$

Substituting for L and T from equations (L) and (T) we have—

$$W = \left[\cdot 0051\lambda k_{L\,max} S\left(1 + \frac{\tan\theta}{L/D}\right) + \frac{R\tan\theta}{10^4}\right]\sigma V'^2.$$

Now $$\frac{R}{10^4 a} = \cdot 0051\, k_{L\,max} S$$

$$\therefore\ W = \frac{R\sigma V'^2}{10^4}\left[\frac{\lambda}{a}\left(1 + \frac{\tan\theta}{L/D}\right) + \tan\theta\right]$$

$$\therefore\ V' = \sqrt{\frac{10^4 W}{R\sigma\left[\frac{\lambda}{a}\left(1 + \frac{\tan\theta}{L/D}\right) + \tan\theta\right]}} \quad . \quad . \quad (V')$$

Now the rate of descent $= V'\sin\theta$ and the rate of horizontal travel relative to the ground $= V'\cos\theta - v'$, therefore we have the equation—

$$\tan\phi = \frac{V'\sin\theta}{V'\cos\theta - v'} = \frac{\tan\theta}{1 - \dfrac{v'}{V'\cos\theta}}.$$

Remembering that $\cos\theta$ is approximately equal to unity we can write—

$$\tan\phi = \frac{\tan\theta}{1 - \dfrac{v'}{V'}} \quad . \quad . \quad . \quad (\tan\phi)$$

It will be noticed that σ has not dropped out, so that the "best gliding angle against a given wind" (and the value of λ corresponding to it) is not independent of altitude.

Equations ($\tan\theta$), (a), (V'), and ($\tan\phi$) are in a form amenable to tabular treatment for values of λ from ·1 to 1·0 (see Chapter XII., page 118).

Such a table can be drawn up and then, by plotting, the minimum value of $\tan\phi$ can be found: this is the tangent of the "best gliding angle against the given wind at the given altitude".

It is of interest to note that for a given value of λ, $\tan\theta$ is constant and $V'\sqrt{\sigma}$ is constant.

Hence we see that if v' and σ both vary, but in such a manner

that $v'\sqrt{\sigma}$ remains constant, then tan ϕ is constant for a given value of λ.

That is to say, that if $v'\sqrt{\sigma}$ is kept constant, tan ϕ is a function of λ only, and therefore the minimum value of tan ϕ and the value of λ at which it occurs are both fixed.

Hence in practice we can find the minimum value of tan ϕ and the associated value of λ for a range of values of v in standard density air, then assign any required value to σ and immediately obtain these quantities for a range of values of v' by making $v'\sqrt{\sigma} = v$ for each of the values of v originally chosen.

From this it follows that the work of finding the best gliding angle for a range of altitudes and wind speeds is far less laborious than would appear at first sight.

II. Full Power Flight.

General.—By full power is meant that the throttle is full open, thus allowing the engine to develop its full torque for the altitude in question *unless that would cause the engine to exceed its normal revolutions*, in which case the engine is supposed throttled down to its correct full revolutions. This last case is not considered to belong to Throttled Flight but to Full Power Flight.

Another way of looking at it is to define Full Power Flight as the condition when the engine is developing either its full torque for the altitude in question or its full revolutions, but is not exceeding either.

From the point of view of safety of the engine from damage there would be no objection to the engine developing more than its full torque for the altitude, provided that that did not involve the engine developing more than its full torque for standard density air. Actually, however, as the throttle cannot be more than full open, this case cannot arise.

Top Speed.—Let us consider a typical machine performance curve with a typical pair of propeller performance curves plotted on the same paper, limiting ourselves for the moment to the case of standard density air.

Let V_0 be the speed at which the P_T and P_R curves (found in Chapter III., page 16) intersect, let V_1 be the speed at which the P curve intersects whichever is the lower of the P_T and P_R curves, and let V_2 be the speed at which it intersects the higher.

Then we are confronted with two types of case, illustrated in Fig. 1 and Fig. 2.

Then in the case of Fig. 1 since the throttle cannot be more than full open, the speed V_2 cannot be reached in horizontal flight, and V_1 is the true top speed of horizontal flight in standard density air.

Again, in the case of Fig. 2, though the speed V_2 could be reached by leaving the throttle full open, this is forbidden as it would result in over-running the engine, so again the true top speed of horizontal flight in standard density air is V_1.

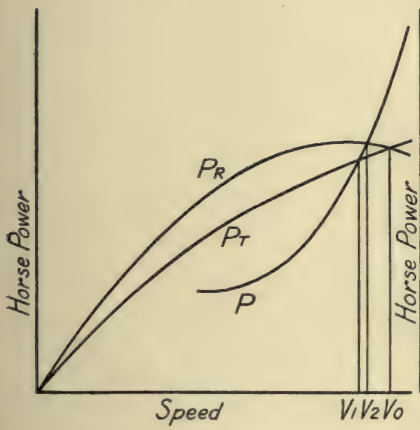


Fig. 1.

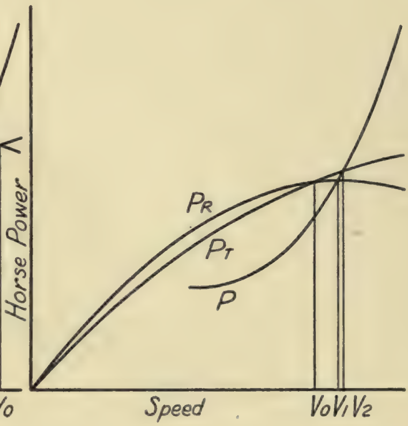


Fig. 2.

Top Speed at an Altitude.—In finding the top speed at an altitude two cases occur corresponding to Fig. 1 and Fig. 2, but of course P_R, P_T, P, V_0, V_1, and V_2 are replaced by P_R', P_T', P', V_0' and V_1' and V_2'.

In the case of Fig. 1, the intersection of the P_T' curve and the P' curve gives the top speed at the altitude V_1'.

In the case of Fig. 2, however, V_1', the intersection of the the P_R' and P' curves is the top speed at the altitude.

Racing Machines.—In the special case of a racing machine, *i.e.* a machine for which the propeller has been specially designed to give the maximum possible top speed in standard density air, we have the case of top speed $= V_0$.

The method of finding V_0 for this case will be given in Chapter X., page 105; this value of V_0 is also the top speed in standard density air in this special case.

All the above work on Full Power Flight takes, of course, the same form, whatever method of finding the machine performance curve has been used. The accuracy of the results, however, naturally depend on the accuracy of the method used in the first place.

Rate of Climb.—*First Approximation.*—If we neglect the influence of the obliquity of the propeller thrust, the fact that the weight is not perpendicular to the direction of flight, and the additional slip stream action due to the use of full power, we can take the horse-power required for mere flight from the machine performance curve calculated by the Third Method, and the horse-power available for mere flight plus climb from either the P_T or the P_R curve, whichever has the lower value at the speed in question (see, for instance, Fig. 1 and Fig. 2, page 41).

The difference between the propeller power and the machine power is the horse-power available for climbing, and the rate of climb in feet per minute is this difference, multiplied by 33,000 and divided by the weight of the machine in pounds.

Corresponding to each speed between the stalling speed under power and the top speed, there is a definite rate of climb: at some particular speed the rate of climb is a maximum, and this maximum is often spoken of simply as "the climb," while the particular speed is often called "the climbing speed".

The above method applies as it stands to any altitude, merely using the curves for P', P_T' and P_R' plotted on V', instead of the curves of P, P_T, and P_R plotted on V.

For rough work the machine performance curve can be taken from the First Method instead of from the Third Method, but in that case, of course, the inaccuracy is increased.

Second Approximation.—This approximation, though still not perfect, goes a stage further than the First Approximation. In it we assume that the forces operating relative to the flight path are the same as for horizontal flight, except that we take account of the propeller thrust being inclined to the horizon (being in fact along the flight path), and of the weight being inclined to the normal to the flight path (being in fact, vertical). That is to say, we assume that the air download on the tail and the propeller slip stream effect (if the Third Method or the Fourth Method is being followed) are unaltered by the fact of climbing.

With these assumptions then, let θ be the inclination of the path to the horizontal, let P be (as usual) the machine horse-power required for *horizontal* flight, let P_P be the propeller horse-power available at the speed in question (then P_P equals P_T or P_R, whichever is the least at the speed in question), and let V be the speed in miles per hour along the path.

Further, let W be (as usual) the total weight of the machine in pounds, let T be the thrust of the propeller in pounds, let L be the actual air lift at right angles to the path of flight in pounds, and let D be the actual total air resistance of the machine along the flight path in pounds.

Then by resolving along and perpendicular to the flight path we have the equations—

$$T = D + W \sin\theta \qquad . \quad . \quad . \quad (1)$$

and

$$L = W \cos\theta \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Also of course—

$$P_P = \frac{TV}{375} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and

$$P = \frac{DV}{375}\left(\frac{W}{L}\right) \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The term $\left(\frac{W}{L}\right)$ in the last equation is due to the fact that on climb L is not the same as W, while it is, at least very approximately, in horizontal flight, to which P refers by the definition we have given it.

Now let $\qquad x = \tan\left(\frac{\theta}{2}\right)$

$$\therefore \cos\theta = \cos 2\left(\frac{\theta}{2}\right) = \frac{\cos^2\left(\frac{\theta}{2}\right) - \sin^2\left(\frac{\theta}{2}\right)}{\cos^2\left(\frac{\theta}{2}\right) + \sin^2\left(\frac{\theta}{2}\right)} = \frac{1 - \tan^2\left(\frac{\theta}{2}\right)}{1 + \tan^2\left(\frac{\theta}{2}\right)}$$

$$= \frac{1 - x^2}{1 + x^2} \quad . \quad . \quad . \quad . \quad (\cos\theta)$$

$$\text{and} \quad \sin\theta = \sin 2\left(\frac{\theta}{2}\right) = \frac{2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{\cos^2\left(\frac{\theta}{2}\right) + \sin^2\left(\frac{\theta}{2}\right)} = \frac{2\tan\left(\frac{\theta}{2}\right)}{1 + \tan^2\left(\frac{\theta}{2}\right)}$$

$$= \frac{2x}{1 + x^2} \quad . \quad . \quad . \quad . \quad (\sin\theta)$$

Now from equations (4) and (2) we have—

$$D = \frac{375PL}{VW} = \frac{375P\cos\theta}{V} \quad . \quad . \quad . \quad (D)$$

Then from equations (1), (3), and (D) we have—

$$\frac{375P_P}{V} = T = D + W\sin\theta = \frac{375P\cos\theta}{V} + W\sin\theta.$$

From this and equations ($\cos\theta$) and ($\sin\theta$) we have—

$$\frac{375P_P}{V} = \frac{375P(1-x^2)}{V(1+x^2)} + \frac{2Wx}{1+x^2}$$

$$\therefore (x^2+1)\frac{375P_P}{V} + (x^2-1)\frac{375P}{V} - 2Wx = 0$$

$$\therefore \frac{375(P_P+P)}{V}x^2 - 2Wx + \frac{375(P_P-P)}{V} = 0.$$

This is a quadratic equation for x, *i.e.* for $\tan\left(\frac{\theta}{2}\right)$, of which the solution is—

$$\tan\left(\frac{\theta}{2}\right) = \frac{WV}{375(P_P+P)} - \sqrt{\left\{\frac{WV}{375(P_P+P)}\right\}^2 - \frac{P_P-P}{P_P+P}}.$$

The angle of climb θ being now determined, we have, if C is the rate of climb in feet per minute—

$$C = V\sin\theta\frac{5280}{60} = 88V\sin\theta$$

$$\therefore C = \frac{176Vx}{1+x^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (C)$$

$$\text{where } x = \frac{WV}{375(P_P+P)} - \sqrt{\left\{\frac{WV}{375(P_P+P)}\right\}^2 - \frac{P_P-P}{P_P+P}}. \quad (x)$$

Equations (C) and (x) are in a form convenient for tabular treatment (see Chapter XII., page 122).

The rate of climb at an altitude is obtained in an exactly similar way, merely writing V', P_P', and P' for V, P_P, and P.

This concludes the investigation of rate of climb under the assumptions of the Second Approximation. This approximation does not differ materially from the First Approximation except when the machine has a really high rate of climb.

Third Approximation.—The assumptions made in this case are that the air download on the tail and the height of the propeller axis may be disregarded, but the full power slip stream is taken account of. Any attempt at a frontal attack on the

problem is doomed to lead to equations only soluble by trial and error. This process, though most useful when none other is available, is terribly laborious, and we should not, therefore, be justified in employing it merely for the purpose of getting a closer approximation than we have yet got.

We will therefore employ a flank attack, making use of the principle that corrections on corrections are unimportant.*

We will start with the machine performance curve obtained by the First Method and with our pair of propeller performance curves.

Now we have to correct this machine performance curve for the full power slip stream: we shall do so on the assumption that the extra lift and the extra resistance can be added (causing the speed to drop and the value of P to be modified, and thus giving us a point on the corrected curve for horizontal flight) *without altering the value of* λ *and thus invalidating the work*: this assumption is untrue, but the error involved, being an error on a correction, need not trouble us.

Well then we have our propeller curves, which give us the value of P_p for any derived value of V, and we have our original curve giving P in terms of V.

First choose a value of λ and then obtain from the table the corresponding value of V and call it ${}_1V$; also get from the table ${}_1P$, the corresponding value of P, and from the curve, ${}_1P_p$, the corresponding value of P_p.

Then the full power thrust T is given by the equation—

$$ {}_1P_p = \frac{{}_1VT}{375}. $$

Hence the slip stream velocity $(1 + b){}_1V$ is given by the equation—

$$ T = \frac{1{\cdot}39}{10^5} d^2 {}_1V^2 B $$

where $$ B = (1 + b)^2 - 1, $$

as we see by referring to page 26, for instance.

Now the total lift under slip stream conditions is—

$$ {\cdot}0051 k_L (S + BS')\,{}_1V^2 $$

(see page 26), whereas under conditions of no slip stream it is—

$$ {\cdot}0051 k_L S {}_1V^2. $$

Therefore, assuming as we do that the value of λ is unaltered,

* This principle, so valuable to the computer, is a bold generalisation to finite differences of the mathematician's principle of neglecting infinitesimals of the second order.

we have that the added lift L, due to introducing the full power slip stream, is given by—

$$\begin{aligned} L &= \cdot 0051 k_L S'_1 V^2 B \\ &= \cdot 0051 k_L S'_1 V^2 \frac{10^5 T}{1 \cdot 39 d^2{}_1V^2} \\ &= \frac{510}{1 \cdot 39} k_L S' \frac{375{}_1P_P}{d^2{}_1V} \\ &= \frac{137{,}500 S' k_{Lmax}}{d^2} \cdot \frac{\lambda_1 P_P}{{}_1V} \quad \quad (L) \end{aligned}$$

Hence L is known for the assumed value of λ.

This additional lift is equivalent to an equal reduction of W and therefore, assuming as we do the value of λ to be unaltered, the machine, instead of flying at ${}_1V$ at this value of λ will really fly at a speed ${}_2V$ given by the equation—

$$ {}_2V = {}_1V \sqrt{\frac{W - L}{W}} \quad \quad ({}_2V)$$

Again, the total resistance under slip stream conditions is—

$$(BR_1 + R_1 + R_2)\left(\frac{{}_1V}{100}\right)^2 + \frac{W + L}{L/D}$$

(compare, for instance, page 26), whereas under conditions of no slip stream it is—

$$(R_1 + R_2)\left(\frac{{}_1V}{100}\right)^2 + \frac{W}{L/D}.$$

Therefore, the added resistance R due to introducing the full power slip stream is given by—

$$\begin{aligned} R &= \frac{BR_1{}_1V^2}{10^4} + \frac{L}{L/D} \\ &= \frac{10}{1 \cdot 39} \cdot \frac{TR_1}{d^2} + \frac{L}{L/D} \\ &= \frac{10}{1 \cdot 39} \frac{R_1}{d^2} 375 \frac{{}_1P_P}{{}_1V} + \frac{L}{L/D} \\ &= 2700 \frac{R_1}{d^2} \frac{{}_1P_P}{{}_1V} + \frac{L}{L/D} \quad \quad (R) \end{aligned}$$

But this is on the assumption that the speed ${}_1V$ is maintained, whereas, as we have seen, this speed is actually reduced to ${}_2V$. Therefore the horse-power required by the machine is really altered to—

$$ {}_2P = \left({}_1P + \frac{R\,{}_1V}{375}\right)\left(\frac{{}_2V}{{}_1V}\right)^3$$

$$= \left({}_1\mathrm{P} + \frac{R\,{}_1\mathrm{V}}{375}\right)\frac{\mathrm{W} - L}{\mathrm{W}}\sqrt{\frac{\mathrm{W} - L}{\mathrm{W}}} \quad . \quad . \quad ({}_2\mathrm{P})$$

The procedure to be adopted therefore is to find ${}_2\mathrm{V}$ and ${}_2\mathrm{P}$ by a tabular process from the above (see Chapter XII., page 123), plot the modified machine performance curve so obtained, and apply to it either the method of the First Approximation or the Second Approximation.

Comparison of the Results of the Three Approximations.—A reference to examples (6), (7), and (8), Chapter XIX., page 169, shows that the rates of climb in feet per minute in standard density air for a particular machine come out as 758, 744, and 707, while the corresponding speeds in miles per hour are 71, 74, and 70 respectively.

We see therefore that, except in unusual cases, the method of the First Approximation, though optimistic, is not far off the mark. This is the method ordinarily used, and it is advisable to retain the other methods for special cases only.

Times to Altitudes.—*General Method.*—In the general case we must first determine C′, the rate of climb in feet per minute at a series of altitudes, from the ground* up to the "ceiling," that is to say, the altitude at which the rate of climb is zero.

Let t be the time in minutes from leaving the ground, and let x_0 be the height of the ground in feet.*

Let x be any altitude in feet on the same basis, then

$$\mathrm{C}' = \frac{dx}{dt}$$

$$\therefore \frac{dt}{dx} = \frac{1}{\mathrm{C}'}$$

$$\therefore dt = \frac{dx}{\mathrm{C}'}$$

$$\therefore t = \int_{x_0}^{x} \frac{dx}{\mathrm{C}'}.$$

Hence the time to any required altitude is readily obtained by plotting $\frac{1}{\mathrm{C}'}$ on a base of standard altitude and then integrating graphically.

* The ground is not the altitude at which σ is zero: two corrections are necessary, one for climate and the other for the height of the ground above sea level. Thus, for instance, for a place at sea level in England under normal conditions the *ground level* may be taken as zero, but the value of σ is not 1 but 1·025 (see curve of page 104), while for the same place in the summer, the ground level must be taken as 2350 and consequently the value of σ as ·952 (see footnote, page 117).

The Particular Case where the Rate of Climb is a Linear Function of the Altitude.—It often happens that when C′ is plotted on x the curve is, at least very approximately, a straight line. In this case let any altitude from ground level in feet be a (then $x = a + x_0$). Then C′ is a linear function of a.

Let the value of C′ at $a = 0$ be c and let the value of a at $C' = 0$ be a_1. Then—

$$C' = c\frac{a_1 - a}{a_1}.$$

Also we have, much as before—

$$\frac{da}{dt} = C'$$

$$\therefore\ t = \int_0^a \frac{da}{C'}$$

$$= \int_0^a \frac{a_1 da}{c(a_1 - a)} = -\frac{a_1}{c}\int_0^a \frac{d(a_1 - a)}{(a_1 - a)}$$

$$= -\frac{a_1}{c}\Big[\log_e (a_1 - a)\Big]_0^a = -\frac{a_1}{c}[\log_e (a_1 - a) - \log_e a_1]$$

$$= -\frac{a_1}{c}\log_e\left(\frac{a_1 - a}{a_1}\right) = \frac{a_1}{c}\log_e\left(\frac{a_1}{a_1 - a}\right)$$

$$= \frac{2{\cdot}303a_1}{c}\log_{10}\left(\frac{a_1}{a_1 - a}\right).$$

Therefore, in this special case (which is of fairly frequent occurrence) we can dispense with the graphical integration and use the above formula instead.

Ceiling.—There is really a "ceiling," *i.e.* a height at which the climb is zero, corresponding to each point on the machine performance curve. Ordinarily the word "ceiling" is applied only to the best of these values, or "maximum ceiling". In what follows, however, we shall refer to a ceiling for each point on the machine performance curve.

Consider a point on the machine performance curve at an altitude which is the ceiling for this point: then the climb is zero, therefore $P_T' = P'$ * at this altitude and at this speed V′.

Now the point P′, V′ was obtained from a particular point P, V on the machine performance curve for standard density air,

* If the propeller were designed with an excessively small value of V_0, the criterion would be $P_R' = P'$, but such a case never occurs in practice, as it would indicate a propeller design which always sacrificed a lot of the available horse-power.

and we see from Chapter IV., page 34, that the two points are related by the equations—

$$V' = \frac{1}{\sqrt{\sigma}} V \quad . \quad . \quad . \quad . \quad (1)$$

and

$$P' = \frac{1}{\sqrt{\sigma}} P \quad . \quad . \quad . \quad . \quad (2)$$

Now we may consider the point P′, V′ as being on the constant torque propeller performance curve at the altitude: for this purpose we will call it P_T', V_T': corresponding to it in standard density air there is a point P_T, V_T, related to it by the equations—

$$V_T' = V_T\sqrt{\frac{\sigma_1}{\sigma}} \quad . \quad . \quad . \quad . \quad (3)$$

and

$$P_T' = P_T\sigma_1\sqrt{\frac{\sigma_1}{\sigma}} \quad . \quad . \quad . \quad . \quad (4)$$

as we see from Chapter III., page 19.

Also, as we have indicated above—

$$V_T' = V' \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and

$$P_T' = P' \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$\therefore \; P_T = P_T'\frac{\sqrt{\sigma}}{\sigma_1\sqrt{\sigma_1}} \quad \text{from (4)}$$

$$= P'\frac{\sqrt{\sigma}}{\sigma_1\sqrt{\sigma_1}} \quad \text{from (6)}$$

$$= \frac{P}{\sqrt{\sigma}}\frac{\sqrt{\sigma}}{\sigma_1\sqrt{\sigma_1}} \quad \text{from (2)}$$

$$\therefore \; \frac{P_T}{P} = \frac{1}{\sigma_1\sqrt{\sigma_1}} \quad . \quad . \quad . \quad . \quad \left(\frac{P_T}{P}\right)$$

Again,

$$V_T = V_T'\frac{\sqrt{\sigma}}{\sqrt{\sigma_1}} \quad \text{from (3)}$$

$$= V'\frac{\sqrt{\sigma}}{\sqrt{\sigma_1}} \quad \text{from (5)}$$

$$= \frac{V}{\sqrt{\sigma}}\frac{\sqrt{\sigma}}{\sqrt{\sigma_1}} \quad \text{from (1)}$$

$$\therefore \; \frac{V_T}{V} = \frac{1}{\sqrt{\sigma_1}} \quad . \quad . \quad . \quad . \quad \left(\frac{V_T}{V}\right)$$

Comparing equations $\left(\frac{P_T}{P}\right)$ and $\left(\frac{V_T}{V}\right)$ we see that

$$\frac{P_T}{P} = \left(\frac{V_T}{V}\right)^3.$$

Therefore, given the point P, V, we can find the point P_T, V_T by plotting through the point P, V a curve of the form

$$y \propto x^3.$$

Starting, therefore, from the point P, V, we can consider the point P_T, V_T as known, since it is the intersection of the curve of the form $y \propto x^3$ which passes through P, V with the P_T curve.

Therefore, V and V_T being known, σ_1 is got from equation $\left(\frac{V_T}{V}\right)$ in the form

$$\sigma_1 = \left(\frac{V}{V_T}\right)^2 \quad . \quad . \quad . \quad (\sigma_1)$$

Now, by the definition of σ_1 given on page 18, we see that

$$\sigma_1 = \frac{\sigma - p}{q}$$

$$\therefore \sigma = q\sigma_1 + p \quad . \quad . \quad . \quad (\sigma)$$

where p and q have the values given on page 19.

Equations (σ) and (σ_1), with the aid of the device of plotting the $y \propto x^3$ curve, determine σ, from which the ceiling is found corresponding to the original point P, V with the aid of the curve of page 104.

The maximum ceiling is found by repeating the work for a few points and finding the maximum in the ordinary way.

A convenient method of applying the theory will be found in Chapter XII., page 125.

III. Throttled Flight.

Slowest Flying Speed.—The slowest flying speed differs from the landing speed on glide, owing to the influence of the propeller slip stream on the wing lift.

The slowest flying speed can be read off the machine performance curve if this has been calculated by the Third Method or the Fourth Method, or it can be obtained by working the performance calculation by the Third Method for the case of $\lambda = 1{\cdot}0$ as far as the determination of V.

Throttling Curves.—In calculations connected with throttling it is often necessary to plot a number of curves of the form $y \propto x^3$ across the machine and propeller performance curves. To avoid the labour of doing this repeatedly it is best to scribe

a complete family of these curves on celluloid and keep it for use when required. It is simply laid over the performance curves with the axes corresponding, and then the intersection of any of the "Throttling Curves," as these cubics are called, with the P, P_T, and P_R curves, can be read off at once. Particulars of how to make such a family of curves on celluloid are given in Chapter XII., page 126.

These throttling curves are only referred to here in order to draw attention to a particularly useful property of the family.

Let one of the curves be

$$y = ax^3$$

and suppose that it has been plotted on inch squared paper. Then suppose that this curve is laid over a performance calculation which is plotted on millimetre squared paper, or in French units, or with an open velocity scale and a close horse-power scale, and let us consider what the meaning of the curve will be when read off on the scale of the paper underneath it.

A moment's consideration will show that the curve will then be

$$\text{horse-power} = \beta\,(\text{velocity})^3,$$

where β is a new constant depending on the scales used in the two directions.

Since, however, β is a *constant*, the curve is still of the form $y \propto x^3$ on the scale of the paper.

Therefore, a set of such curves, once scribed on celluloid, can be used on any kind of plotting of horse-power on a base of speed, *irrespective of the units used and the scales chosen.*

Consumption and Revolutions when Throttled.—Aero engines will not run indefinitely at full power without taking harm, and for this reason it is desirable to know both the consumption and the revolutions when flying horizontally, throttled down to any speed.*

Also it is preferred that a commercial machine should, for reasons of economy, normally travel throttled: to investigate this, again, we need to be able to calculate the consumption at any speed and at any height.

We will therefore find the revolutions and the indicated horse-power when the machine is flying horizontally at a given speed at a given height—throttled of course.

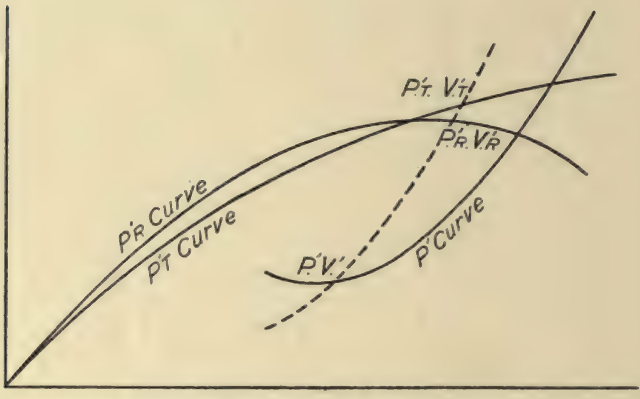
* As an example, the makers of one engine advise that if it is used when cruising at not more than ·7 of full consumption, and at not more than ·9 of full revolutions, its life will be practically indefinitely prolonged.

P' is the effective horse-power required by the machine for horizontal flight at the altitude in question.

P_T' is the effective horse-power given by the propeller at full torque for the altitude.

P_R' is the effective horse-power given by the propeller at full revolutions.

Now suppose that we have curves of P', P_T', and P_R' plotted on a base of speed, and let us consider three particular points (one on each of the three curves), whose co-ordinates are P', V', P_T', V_T', and P_R', V_R' respectively. Also we will take the following additional definitions :—



I' is the indicated horse-power.

F' is the horse-power lost in friction in the engine.

H' is the brake horse-power.

Q' is the indicated torque.

N' is the engine revolutions.

T' is the thrust.

The above refer to the conditions when the machine is flying horizontally throttled, *i.e.* they refer to the point P', V'.

Corresponding letters with suffix T refer to the point P_T', V_T'.

Corresponding letters with suffix R refer to the point P_R', V_R'.

Corresponding letters in the italic character refer to the point of intersection of the curves P_T' and P_R', *i.e.* to the condition when

the engine is developing *full revolutions* and *full indicated torque for the altitude* (which we already know from Chapter III., page 19, to be σ times the full indicated torque in standard density air).

Now further, let P_T', V_T' and P_R', V_R' be so related to P', V' that

$$\frac{N_T'}{N'} = \frac{V_T'}{V} \quad . \quad . \quad . \quad . \quad (1)$$

and
$$\frac{N_R'}{N'} = \frac{V_R'}{V'} \quad . \quad . \quad . \quad . \quad (2)$$

Then it follows from propeller theory that

$$\frac{T_T'}{T'} = \left(\frac{V_T'}{V'}\right)^2 \quad . \quad . \quad . \quad . \quad (3)$$

and
$$\frac{T_R'}{T'} = \left(\frac{V_R'}{V'}\right)^2 \quad . \quad . \quad . \quad . \quad (4)$$

Also, of course,
$$\frac{P_T'}{P'} = \frac{T_T'V_T'}{T'V'} \quad . \quad . \quad . \quad . \quad (5)$$

and
$$\frac{P_R'}{P'} = \frac{T_R'V_R'}{T'V'} \quad . \quad . \quad . \quad . \quad (6)$$

From (3) and (5) we have

$$\frac{P_T'}{P'} = \left(\frac{V_T'}{V'}\right)^3 \quad . \quad . \quad . \quad . \quad (7)$$

and from (4) and (6) we have

$$\frac{P_R'}{P'} = \left(\frac{V_R'}{V'}\right)^3.$$

That is to say, that the three points we are considering are connected by the fact that a "Throttling Curve" passes through them.

In other words we could first choose P', V' as any point on the P' curve through which a curve of the family passes, and then read off the values of V_T' and V_R' by noting the intersections of this same curve of the family with the P_T' and P_R' curves.

Now
$$N_R' = \mathcal{N} \text{ by definition} \quad . \quad . \quad . \quad (8)$$

$$\therefore \frac{N'}{\mathcal{N}} = \frac{N'}{N_R'} = \frac{V'}{V_R'} \text{ from } (2)$$

$$\therefore N' = \mathcal{N}\left(\frac{V'}{V_R'}\right) \quad . \quad . \quad . \quad . \quad . \quad (N')$$

Again $$\mathrm{I}' = \mathrm{H}' + \mathrm{F}' \quad . \quad . \quad . \quad . \quad . \quad (9)$$

and $$\mathrm{I_T}' = \mathrm{H_T}' + \mathrm{F_T}' \text{ (by definition)} \quad . \quad . \quad (10)$$

$$\frac{\mathrm{P}'}{\mathrm{H}'} = \frac{\mathrm{P_T}'}{\mathrm{H_T}'} \quad . \quad . \quad . \quad . \quad . \quad (11)$$

for reasons of propeller theory from (1)—

$$\mathrm{Q_T}' = Q' \text{ (by definition)} \quad . \quad . \quad . \quad (12)$$

$$Q' = \sigma Q \text{ (from page 53)} \quad . \quad . \quad . \quad (13)$$

$$\frac{\mathrm{F}'}{\mathrm{N}'} = \frac{F}{N} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

and $$\frac{\mathrm{F_T}'}{\mathrm{N_T}'} = \frac{F}{N} \quad . \quad . \quad . \quad . \quad . \quad (15)$$

because the friction losses are proportional to the revolutions and independent of altitude. Also, of course,

$$\frac{\mathrm{I_T}'}{\mathrm{N_T}'\mathrm{Q_T}'} = \frac{I}{NQ} \quad . \quad . \quad . \quad . \quad (16)$$

The letters in italics without dashes refer, of course, to the full revolutions and full torque condition in standard density air.

Now from the above we deduce—

$$\frac{\mathrm{I}'}{I} = \frac{\mathrm{H}' + \mathrm{F}'}{I} \text{ from (9)}$$

$$= \frac{\mathrm{H_T}'\mathrm{P}'}{I\mathrm{P_T}'} + \frac{F\mathrm{N}'}{IN} \text{ from (11) and (14)}$$

$$= \frac{(\mathrm{I_T}' - \mathrm{F_T}')\mathrm{P}'}{I\mathrm{P_T}'} + \frac{F\mathrm{N}'}{IN} \text{ from (10)}$$

$$= \frac{\mathrm{P}'}{\mathrm{P_T}'}\left[\frac{\mathrm{I_T}'}{I} - \frac{\mathrm{N_T}'F}{NI}\right] + \frac{\mathrm{N}'F}{NI} \text{ from (15)}$$

$$= \frac{\mathrm{P}'}{\mathrm{P_T}'}\left[\frac{\mathrm{N_T}'\mathrm{Q_T}'}{NQ} - \frac{\mathrm{N_T}'F}{NI}\right] + \frac{\mathrm{N}'F}{NI} \text{ from (16)}$$

$$= \frac{\mathrm{P}'}{\mathrm{P_T}'}\left[\sigma\frac{\mathrm{N_T}'}{\mathrm{N_R}'} - \frac{\mathrm{N_T}'F}{\mathrm{N_R}'I}\right] + \frac{\mathrm{N}'F}{\mathrm{N_R}'I} \text{ from (12), (13), and (8)}$$

$$= \frac{\mathrm{P}'}{\mathrm{P_T}'} \cdot \frac{\mathrm{N_T}'}{\mathrm{N}'} \cdot \frac{\mathrm{N}'}{\mathrm{N_R}'}\left[\sigma - \frac{F}{I}\right] + \frac{\mathrm{N}'}{\mathrm{N_R}'} \cdot \frac{F}{I}$$

$$= \left(\frac{\mathrm{V}'}{\mathrm{V_T}'}\right)^3 \frac{\mathrm{V_T}'}{\mathrm{V}'} \cdot \frac{\mathrm{V}'}{\mathrm{V_R}'}\left[\sigma - \frac{F}{I}\right] + \frac{\mathrm{V}'}{\mathrm{V_R}'} \cdot \frac{F}{I} \text{ from (7), (1), and (2)}$$

$$= \frac{\mathrm{V}'}{\mathrm{V_R}'}\left[\frac{F}{I} + \left(\sigma - \frac{F}{I}\right)\left(\frac{\mathrm{V}'}{\mathrm{V_T}'}\right)^2\right]$$

$$\therefore\ \mathrm{I}' = I\left\{p + (\sigma - p)\left(\frac{\mathrm{V}'}{\mathrm{V}_{\mathrm{T}}'}\right)^2\right\}\frac{\mathrm{V}'}{\mathrm{V}_{\mathrm{R}}'} \quad . \quad . \quad (\mathrm{I}')$$

the definitions of p and q, and their numerical values, are given on page 19.

Equations (N′) and (I′) give for any speed and altitude the revolutions and the ratio of the consumption to full consumption, after V_{T}' and V_{R}' have been found from the value of V′ by the use of the "Throttling Curves".

Best Cruising Speed at a Given Altitude.—The consumption per mile is proportional to the consumption per hour divided by the speed, and, as we see from the last paragraph, the consumption per hour is proportional to

$$\left\{p + (\sigma - p)\left(\frac{\mathrm{V}'}{\mathrm{V}_{\mathrm{T}}'}\right)^2\right\}\frac{\mathrm{V}'}{\mathrm{V}_{\mathrm{R}}'}$$

therefore the consumption per mile is proportional to

$$\frac{p + (\sigma - p)\left(\dfrac{\mathrm{V}'}{\mathrm{V}_{\mathrm{T}}'}\right)^2}{\mathrm{V}_{\mathrm{R}}'}.$$

We can now take a few values of V′, find the above quantity for each, and plot to find the minimum and note the speed at which it occurs.

This is the best cruising speed at that altitude for flight in still air.

For cruising against a head wind at the altitude of speed v', we get the consumption per mile made good over the map is proportional to

$$\frac{\mathrm{V}'}{\mathrm{V}_{\mathrm{R}}'(\mathrm{V}' - v')}\left\{p + (\sigma - p)\left(\frac{\mathrm{V}'}{\mathrm{V}_{\mathrm{T}}'}\right)^2\right\}$$

which again can be plotted to find the best conditions.

Best Cruising Altitude.—The best cruising altitude can be found by comparing the best consumptions per mile at a range of altitudes. In practice, however, the cruising altitude is limited by navigation problems, so that the calculation, if made, has perhaps only an academic interest. The influence of altitude on cruising with and against the wind can also be investigated at the same time.

Long Range Cruising.—By long range cruising is meant cruising over such great distances that the weight of the fuel used is a very serious item, requiring careful attention in order to get the most out of a gallon.

The results obtained by this investigation can, of course, be applied to ordinary journeys and should make it easier to maintain good economy: their real application, however, is to such a problem as "Against how fast a head wind will the machine in question cross the Atlantic?"

We will suppose the flight to take place at constant altitude, so that σ is constant.

Also we will disregard head winds at first, and allow for them at the end.

We have from page 55

$$I' = I\left\{p + (\sigma - p)\left(\frac{V'}{V_T'}\right)^2\right\}\frac{V'}{V_R'}.$$

Also we will take the following definitions:—

x is the distance in miles covered from the start up to the moment under consideration.

t is the time *in hours* from the start.

W is the total weight of the machine at time t. Note that W is a variable, owing to the consumption of fuel.

δ is the total pounds *per hour* of fuel being expended at the moment under consideration. Note this includes petrol, oil, and, in the case of water-cooled engines, that part of the total water supply that is evaporated.

N' is the revolutions (as usual).

N is the full revolutions (as usual).

Δ is the full consumption in standard density air.

Suffix o refers to the commencement of the cruise.

We now assume that the pilot reduces speed as the machine gets lighter so as to always fly at the same value of λ (as a matter of fact all he has to do is to leave the elevator setting alone): of course, he will also have to throttle down progressively in order to avoid climbing above the constant altitude.

Since λ is constant, the "gliding angle" is constant, and of course we have—

$$\left(\frac{V'}{V_0'}\right)^2 = \frac{W}{W_0} \quad . \quad . \quad . \quad . \quad (1)$$

and

$$\frac{P'}{P_0'} = \frac{WV'}{W_0V_0'}$$

$$\therefore \frac{P'}{P_0'} = \left(\frac{V'}{V_0'}\right)^3$$

$\therefore$ P', V' is always on the same "Throttling Curve";
but V_T' and V_R' are on the same "Throttling Curve" as V',

hence V_T' and V_R' are constant and equal to V_{T0}' and V_{R0}' respectively. Then we have—

$$\frac{\delta}{\Delta} = \frac{I'}{I} = \frac{V'}{V_R'}\left\{p + (\sigma - p)\left(\frac{V'}{V_T'}\right)^2\right\} \quad . \quad . \quad (2)$$

and from page 53

$$\frac{N'}{N} = \frac{V'}{V_R'} \quad . \quad . \quad . \quad . \quad (3)$$

Also, of course, by definition

$$\delta = -\frac{dW}{dt} \quad . \quad . \quad . \quad . \quad (4)$$

and

$$V' = \frac{dx}{dt} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$\therefore$ from (4) and (5)

$$\frac{\delta}{V'} = -\frac{dW}{dx} \quad . \quad . \quad . \quad . \quad (6)$$

$\therefore$ from (2) and (6)

$$-\frac{dW}{dx} = \frac{\Delta}{V_R'}\left\{p + (\sigma - p)\left(\frac{V'}{V_T'}\right)^2\right\}$$

$$= \frac{\Delta}{V_R'}\left\{p + (\sigma - p)\left(\frac{V_0'}{V_T'}\right)^2 \frac{W}{W_0}\right\} \text{ from (1)}$$

$$= \frac{\Delta(\sigma - p)V_0'^2}{W_0 V_R' V_T'^2}\left\{W + \frac{pV_T'^2 W_0}{(\sigma - p)V_0'^2}\right\}.$$

Now put

$$a = \frac{\Delta(\sigma - p)V_0'^2}{W_0 V_R' V_T'^2} \quad . \quad . \quad . \quad . \quad (a)$$

and

$$w = W + \frac{pV_T'^2 W_0}{(\sigma - p)V_0'^2} \quad . \quad . \quad . \quad (w)$$

then

$$\frac{dw}{dx} = \frac{dW}{dx}$$

$$\therefore \; -\frac{dw}{dx} = aw$$

$$\therefore \; dx = -\frac{dw}{aw} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$\therefore \; -\int_{w_0}^{w} \frac{dw}{w} = a\int_{x_0}^{x} dx.$$

Note that $x_0 = 0$ and $w_0 = W_0\left\{1 + \dfrac{pV_T'^2}{(\sigma - p)V_0'^2}\right\}$. (w_0)

Performing the integration, we have—

$$ax = -\Big[\log_e w\Big]_{w_0}^{w} = \Big[\log_e w\Big]_{w}^{w_0} = \log_e\left(\frac{w_0}{w}\right)$$

$$\therefore\ x = \frac{2 \cdot 303}{a}\log_{10}\left(\frac{w_0}{w}\right) \quad . \quad . \quad . \quad . \quad . \quad (x)$$

Again $dt = \dfrac{dx}{V'}$ from (5)

$$= -\frac{dw}{aV'w} \text{ from (7)}$$

$$= -\frac{\sqrt{W_0}\,dw}{\sqrt{W}\,aV_0'w} \text{ from (1)}$$

$$= -\frac{\sqrt{W_0}\,dW}{\sqrt{W}\,aV_0'\left\{W + \dfrac{pV_T'^2W_0}{(\sigma - p)V_0'^2}\right\}} \text{ from } (w) \quad . \quad (dt)$$

Now one of the standard cases of the differential calculus gives us—

$$\frac{dy}{b^2 + y^2} = \frac{1}{b}d\left\{\tan^{-1}\left(\frac{y}{b}\right)\right\}$$

$$\therefore\ \frac{2}{b}d\left\{\tan^{-1}\left(\frac{y}{b}\right)\right\} = \frac{2y\,dy}{y(b^2 + y^2)} = \frac{d(y^2)}{\sqrt{y^2}(b^2 + y^2)} \quad . \quad (8)$$

Now let $y = \sqrt{W}$ . . . . (y)

and $b = \sqrt{\dfrac{pV_T'^2W_0}{(\sigma - p)V_0'^2}}$ . . . . . . (b)

$$\therefore\ dt = -\frac{\sqrt{W_0}}{aV_0'}\,\frac{d(y^2)}{\sqrt{y^2}(y^2 + b^2)} \text{ from } (dt), (y), \text{ and } (b)$$

$$= -\frac{2\sqrt{W_0}}{baV_0'}d\left\{\tan^{-1}\left(\frac{y}{b}\right)\right\}$$

$$\therefore\ \int_0^t dt = -\frac{2\sqrt{W_0}}{baV_0'}\int_{y_0}^{y} d\left\{\tan^{-1}\left(\frac{y}{b}\right)\right\}$$

$$\therefore\ t = -\frac{2\sqrt{W_0}}{baV_0'}\left[\tan^{-1}\left(\frac{y}{b}\right) - \tan^{-1}\left(\frac{y_0}{b}\right)\right]$$

$$= \frac{2\sqrt{W_0}}{baV_0'}\left[\tan^{-1}\left(\frac{y_0}{b}\right) - \tan^{-1}\left(\frac{y}{b}\right)\right] \quad . \quad . \quad (9)$$

Now there is a trigonometrical formula—

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\therefore\ A - B = \tan^{-1}\left\{\frac{\tan A - \tan B}{1 + \tan A \tan B}\right\}.$$

Now let $$A = \tan^{-1}\left(\frac{y_0}{b}\right) \quad . \quad . \quad . \quad . \quad (A)$$

and $$B = \tan^{-1}\left(\frac{y}{b}\right) \quad . \quad . \quad . \quad . \quad (B)$$

$$\therefore\ \tan A = \frac{y_0}{b}$$

and $$\tan B = \frac{y}{b}.$$

$$\therefore\ \left[\tan^{-1}\left(\frac{y_0}{b}\right) - \tan^{-1}\left(\frac{y}{b}\right)\right] = A - B = \tan^{-1}\left\{\frac{\tan A - \tan B}{1 + \tan A \tan B}\right\}$$

$$= \tan^{-1}\left\{\frac{\frac{y_0}{b} - \frac{y}{b}}{1 + \frac{y_0}{b}\frac{y}{b}}\right\}$$

$$= \tan^{-1}\left\{\frac{b(y_0 - y)}{b^2 + y_0 y}\right\} = \tan^{-1}\left\{\frac{b(\sqrt{W_0} - \sqrt{W})}{b^2 + \sqrt{W_0 W}}\right\}$$

$$\therefore\ \text{from (9)}\ t = \frac{2\sqrt{W_0}}{baV_0'}\tan^{-1}\left\{\frac{b(\sqrt{W_0} - \sqrt{W})}{b^2 + \sqrt{W_0 W}}\right\}$$

where the angle is in radians of course.

$$\therefore\ t = \frac{2\sqrt{W_0}}{57{\cdot}3baV_0'}\tan^{-1}\left\{\frac{b(\sqrt{W_0} - \sqrt{W})}{b^2 + \sqrt{W_0 W}}\right\} \quad . \quad . \quad (t)$$

where the angle is in degrees.

Again, from (1) $$V' = V_0'\sqrt{\frac{W}{W_0}} \quad . \quad . \quad . \quad (V')$$

Hence, substituting in (2), we have—

$$\delta = \Delta\frac{V_0'}{V_R'}\sqrt{\frac{W}{W_0}}\left\{p + (\sigma - p)\left(\frac{V_0'}{V_T'}\right)^2\frac{W}{W_0}\right\} \quad . \quad . \quad (\delta)$$

And from (3) and (V′) we have—

$$N' = N\frac{V_0'}{V_R'}\sqrt{\frac{W}{W_0}} \quad . \quad . \quad . \quad . \quad (N')$$

We now have, after the burning of fuel has reduced the total weight of the machine from W_0 to W, the revolutions and rate of consumption given by equations (N′) and (δ).

The proper speed of flight to be maintained at that stage is given by equation (V′).

The air mileage that has been covered up to that stage is given by equation (x) with the help of equations (a) and (w).

The time elapsed on the journey up to that stage is given by equation (t) with the help of equations (a) and (b). We shall need (t) if we wish to know the cruising conditions against a head wind.

CHAPTER VI.

GROUND PERFORMANCE.

General.—Apart from the performance of an aeroplane in the air it is often necessary to know how it will perform on the ground. Thus, for instance, in connection with the size of aerodrome required, the distance required for the machine to come to rest on landing and the length of run to get off the ground are sometimes laid down in a specification: also before the trials of a new type, the pilot may ask to be told these figures.

Again, aeroplanes are often launched from and landed on the decks of ships, so we must calculate the length of deck required in each case.

These matters are dealt with in the present chapter under the general heading of "Ground Performance".

Getting Off a Deck.—Dynamical problems involving relative motion are full of traps for the unwary. It is therefore advised that the line of investigation here given be not departed from, since to do so may give results different from those here found and leave the investigator faced with some fallacy which may be very difficult to locate. The method here given is accepted by those who are competent to judge—and gives close predictions of the results obtained in practice.

In launching a machine from a deck, it will probably be arranged to do its run at about its angle of attack for minimum resistance, and then be "hoiked" up to its angle of attack for maximum lift at the end of the run.

Machines specially designed for deck work often have their propeller shafts inclined so that the propeller slip stream meets the wing at a larger positive angle than is ordinarily the case: this is done in order to reduce as far as possible the *minimum flying speed under full power.*

Let v_m be the minimum flying speed of the machine under full power in *feet per second.* The method of finding v_m for any particular machine will be given later on page 69.

At air speeds between zero and v_m the horizontal component of the thrust of a propeller on full torque, *i.e.* on full throttle, is

very nearly constant. We will assume it to be constant and equal to T *poundals*. The method of finding T will be given later on page 67.

Let KV^2 be the total resistance of the machine (wings and body) at V feet per second relative to the air in *poundals*, including the effect of slip stream action on the wings and body. The method of finding K will be given later on page 67.

Let M be the total mass of the machine in *pounds* (then M is represented by the same number as ordinarily represents W, the weight of the machine in pounds, in other calculations).

Let t be the time in *seconds*, reckoned from the starting of the run.

Let v_s be the forward velocity of the ship in *feet per second*, relative to fixed axes.

Let v_w be the velocity in *feet per second* of the wind as read by an anemometer on the ship. Then the *forward* velocity of the wind relative to the fixed axes is $v_s - v_w$.

Let X be the linear co-ordinate in *feet* of the ship relative to the fixed axes.

Let $X + x$ be the linear co-ordinate of the machine in *feet* relative to the fixed axes. Then x is the co-ordinate of the machine relative to the ship.

Let l be the length of run in *feet* relative to the ship which is required before the machine reaches its minimum flying speed v_m relative to the wind.

Then $$M\frac{d^2(X+x)}{dt^2} = T - KV^2 \quad . \quad . \quad . \quad (1)$$

Now $$\frac{d^2(X+x)}{dt^2} = \frac{d^2X}{dt^2} + \frac{d^2x}{dt^2}$$

and $$\frac{d^2X}{dt^2} = \frac{dv_s}{dt} = 0, \text{ since } v_s \text{ is constant,}$$

$$\therefore \frac{d^2(X+x)}{dt^2} = \frac{d^2x}{dt^2} = \frac{d\dot{x}}{dt} = \frac{dx}{dt}\frac{d\dot{x}}{dx} = \dot{x}\frac{d\dot{x}}{dx} \quad . \quad . \quad (2)$$

Also the speed of the machine relative to the air equals the absolute forward velocity of the machine minus the absolute forward velocity of the air.

$$\therefore V = \frac{d(X+x)}{dt} - (v_s - v_w) = \dot{X} + \dot{x} - v_s + v_w = v_s + \dot{x} - v_s + v_w$$

$$\therefore V = v_w + \dot{x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\therefore \dot{x}\frac{d\dot{x}}{dx} = (V - v_w)\frac{d(V - v_w)}{dx} \text{ from (3)}$$

$$= (V - v_w)\left[\frac{dV}{dx} - \frac{dv_w}{dx}\right]$$

but v_w is constant, $\therefore \dfrac{dv_w}{dx} = 0$

$$\therefore \dot{x}\frac{d\dot{x}}{dx} = (V - v_w)\frac{dV}{dx}$$

$$\therefore dx = \frac{(V - v_w)dV}{\dot{x}\dfrac{d\dot{x}}{dx}}$$

$$= \frac{(V - v_w)dV}{\dfrac{d^2(X + x)}{dt^2}} \text{ from (2)}$$

$$= \frac{M(V - v_w)dV}{T - KV^2} \text{ from (1)}$$

$$= \frac{M}{2}\frac{2VdV}{T - KV^2} - Mv_w\frac{dV}{T - KV^2}$$

$$= \frac{M}{2}\frac{d(V^2)}{T - KV^2} - Mv_w\frac{dV}{T - KV^2}$$

Put $$a = \sqrt{\frac{T}{K}} \quad . \quad . \quad . \quad . \quad (a)$$

$$\therefore dx = -\frac{M}{2K}\frac{d(T - KV^2)}{T - KV^2} - \frac{Mv_w}{K}\frac{dV}{a^2 - V^2}.$$

Now $\dfrac{d(T - KV^2)}{T - KV^2} = d \log_e (T - KV^2).$

And
$$\frac{dV}{a^2 - V^2} = \frac{1}{2a}\left[\frac{1}{a + V} + \frac{1}{a - V}\right]dV$$

$$= \frac{1}{2a}\left[\frac{dV}{a + V} + \frac{dV}{a - V}\right]$$

$$= \frac{1}{2a}\left[\frac{d(a + V)}{a + V} - \frac{d(a - V)}{a - V}\right]$$

$$= \frac{1}{2a}[d \log_e (a + V) - d \log_e (a - V)]$$

$$= \frac{1}{2a}d \log_e \left(\frac{a + V}{a - V}\right)$$

$$\therefore \int_0^l dx = -\frac{M}{2K}\int_{V=v_w}^{V=v_m} d\log_e(T - KV^2) - \frac{Mv_w}{2aK}\int_{V=v_w}^{V=v_m} d\log_e\left(\frac{a+V}{a-V}\right)$$

$$\therefore l = \frac{M}{2K}\log_e\left\{\frac{T - Kv_w^2}{T - Kv_m^2}\right\} + \frac{Mv_w}{2aK}\log_e\left\{\frac{(a+v_w)(a-v_m)}{(a-v_w)(a+v_m)}\right\}$$

$$= \frac{M}{2K}\log_e\left\{\frac{(a+v_w)(a-v_w)}{(a+v_m)(a-v_m)}\right\} + \frac{Mv_w}{2aK}\log_e\left\{\frac{(a+v_w)(a-v_m)}{(a+v_m)(a-v_w)}\right\}$$

$$\therefore l = 1{\cdot}151\,\frac{M}{K}\left[\left(1+\frac{v_w}{a}\right)\log_{10}\left(\frac{a+v_w}{a+v_m}\right) + \left(1-\frac{v_w}{a}\right)\log_{10}\left(\frac{a-v_w}{a-v_m}\right)\right].$$

This equation with the aid of (a) gives the required length of run to get off the deck.

Getting off the Ground.—From this we can get the length run to get off the ground in still air by putting $v_w = 0$: we get

$$l = 1{\cdot}151\,\frac{M}{K}\left[\log_{10}\left(\frac{a}{a+v_m}\right) + \log_{10}\left(\frac{a}{a-v_m}\right)\right].$$

Landing on a Deck.—We will use the same notation as in the previous piece of work, remarking, however, that as the pilot has to approach the deck with caution, v_m, his speed of alighting, will be his minimum flying speed throttled for horizontal flight, *i.e.* his proper "slowest flying speed," which has been discussed on page 50.

Also we now have $T = 0$ and the motion is one of deceleration, not acceleration. Then, following the general method of the previous investigation, we have—

$$M\frac{d^2(X+x)}{dt^2} = -KV^2 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\frac{d^2(X+x)}{dt^2} = \dot{x}\frac{d\dot{x}}{dx} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$V = v_w + \dot{x} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and

$$\dot{x}\frac{d\dot{x}}{dx} = (V - v_w)\frac{dV}{dx}$$

$$\therefore dx = \frac{(V - v_w)dV}{\dot{x}\frac{d\dot{x}}{dx}}$$

$$= \frac{(V - v_w)dV}{\frac{d^2(X+x)}{dt^2}} \text{ from (2)}$$

$$= \frac{M(V - v_w)dV}{-KV^2} \text{ from (1)}$$

$$= -\frac{M}{K}\frac{dV}{V} + \frac{Mv_w}{K}\frac{dV}{V^2}$$

$$= -\frac{M}{K}d\log_e(V) - \frac{Mv_w}{K}d\left(\frac{1}{V}\right)$$

$$\therefore \int_0^l dx = -\frac{M}{K}\int_{V=v_m}^{V=v_w} d\log_e(V) - \frac{Mv_w}{K}\int_{V=v_m}^{V=v_w} d\left(\frac{1}{V}\right)$$

$$\therefore l = \frac{M}{K}\log_e\left(\frac{v_m}{v_w}\right) - \frac{Mv_w}{K}\left\{\frac{1}{v_w} - \frac{1}{v_m}\right\}$$

$$\therefore l = \frac{M}{K}\left[2\cdot303\log_{10}\left(\frac{v_m}{v_w}\right) - \frac{v_m - v_w}{v_m}\right] \quad . \quad (l)$$

Equation (l) gives the required length of deck for landing on a ship.

From this we can get the length of run for landing on an aerodrome in still air, by putting $v_w = 0$.

We see that it leads to $l = \infty$; that is to say, that if there was no ground friction, the air resistance would never quite stop the machine moving. We must therefore investigate the length of landing run, taking account of friction.

Landing on the Ground.—We suppose that brakes on the wheels, the friction of the tail skid, or even simply the friction of the ordinary wheel axles, have the effect that the machine can be considered to have a coefficient of friction μ with the ground.

Let M be the mass of the machine in *pounds*.

Let x be the distance in *feet* from the point of contact with the ground.

Let v be the landing speed in *feet per second.* This is either the "slowest flying speed," which has been found on page 50, or the "landing speed on glide," which has been found on page 35, according to whether the pilot lands with engine on or off.

Let t be the time in *seconds* after touching the ground.

Let KV^2 be the total resistance of wings and body at the altitude at which the machine runs on the ground in *poundals* at speed V feet per second.

Let l be the length of run in *feet.*

Then the air-borne weight $= Mg\frac{\dot{x}^2}{v^2}$

$\therefore$ the ground-borne weight $= \mathrm{M}g\left(1 - \frac{\dot{x}^2}{v^2}\right)$

$\therefore$ the equation of motion is—

$$\mathrm{M}\ddot{x} = -\mathrm{K}\dot{x}^2 - \mu \mathrm{M}g\left(1 - \frac{\dot{x}^2}{v^2}\right)$$

$$\therefore \ddot{x} + \left(\frac{\mathrm{K}}{\mathrm{M}} - \frac{\mu g}{v^2}\right)\dot{x}^2 + \mu g = 0 \qquad . \quad . \quad (1)$$

Now $$\ddot{x} = \frac{d\dot{x}}{dt} = \frac{d\dot{x}}{dx}\frac{dx}{dt} = \dot{x}\frac{d\dot{x}}{dx} = \frac{d(\dot{x}^2)}{2dx}$$

$\therefore$ from (1) $$\frac{d(\dot{x}^2)}{dx} + 2a\dot{x}^2 + 2\mu g = 0 \,. \qquad . \quad . \quad (2)$$

where $$a = \frac{\mathrm{K}}{\mathrm{M}} - \frac{\mu g}{v^2} \quad . \quad . \quad . \quad . \quad (a)$$

Now we get different formulæ according to whether a does or does not equal zero. First, then, if $a = 0$, we have from (2)

$$\frac{d(\dot{x}^2)}{dx} + 2\mu g = 0$$

$$\therefore \int_{\dot{x}=v}^{\dot{x}=0} d(\dot{x}^2) + 2\mu g\int_0^l dx = 0$$

$$\therefore -v^2 + 2\mu g l = 0$$

$$\therefore l = \frac{v^2}{2\mu g} \quad . \quad . \quad . \quad . \quad . \quad (l_1)$$

But if a is not zero, we have—

$$d(a\dot{x}^2 + \mu g) = a\,d(\dot{x}^2)$$

$\therefore$ from (2) $$\frac{d(a\dot{x}^2 + \mu g)}{a} + 2(a\dot{x}^2 + \mu g)dx = 0$$

$$\therefore \int_{\dot{x}=v}^{\dot{x}=0} \frac{d(a\dot{x}^2 + \mu g)}{a\dot{x}^2 + \mu g} + 2a\int_0^l dx = 0$$

$$\therefore \int_{\dot{x}=v}^{\dot{x}=0} d\log_e(a\dot{x}^2 + \mu g) + 2a\int_0^l dx = 0$$

$$\therefore \log_e(\mu g) - \log_e(av^2 + \mu g) + 2al = 0$$

$$\therefore l = \frac{1}{2a}\log_e\left(\frac{av^2 + \mu g}{\mu g}\right)$$

but from (a) $$av^2 = \frac{\mathrm{K}}{\mathrm{M}}v^2 - \mu g$$

$$\therefore\ l = \frac{1}{2a}\log_e\left(\frac{\frac{K}{M}v^2}{\mu g}\right)$$

$$\therefore\ l = \frac{1 \cdot 151}{a}\log_{10}\left(\frac{Kv^2}{M\mu g}\right) \quad . \quad . \quad . \quad (l_2)$$

Equation (l_1) or (l_2) gives the length of landing run, according to whether $a = 0$ or not.

In practice equation (l_2) would give poor accuracy if a was nearly equal to 0: in that case, therefore, equation (l_1) should be taken instead as an approximation.

Propeller Thrust at Slow Speeds.—For speeds between zero and the getting off speed, a reference to the figures on page 41, for instance, will show that the effective horse-power P_T is nearly proportional to the speed V. Therefore, the thrust is nearly constant, though it is higher, slightly, at lower speeds.

Let V be a speed in *miles per hour* somewhere round about three-quarters of the estimated getting off speed—never mind details.

Let P_T be the effective horse-power from the P_T curve at this value of V.

Then the thrust in *pounds* $= \frac{375P_T}{V}$, and therefore T, the thrust in *poundals*, is given by—

$$T = \frac{32 \cdot 2 \times 375P_T}{V}.$$

Total Resistance.—If the machine is not designed with the propeller shaft far from horizontal when getting off, we can employ the Third Method of machine performance calculation, remembering, however, that the thrust to which the slip stream is due is the full thrust just calculated.

Using the same general line of investigation as on page 25, and using the *same units as there defined*, but simplifying the work by omitting to take account of l, l', and k_c, we have—

$$K'V^2 = [(1 + b)^2R_1 + R_2]\left(\frac{V}{100}\right)^2 + \frac{W}{L/D} \quad . \quad . \quad (1)$$

corresponding to equation (2) of page 25;

$$W = \cdot 00237k_L[S - S' + (1 + b)^2S'](1 \cdot 467V)^2 \quad . \quad (2)$$

corresponding to equation (3), page 25;

$$k_L = \lambda k_{L max} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

corresponding to equation (5), page 25; and

$$T' = \frac{\cdot 00237}{2}\frac{\pi}{4}\left(\frac{d}{12}\right)^2(1\cdot 467V)^2 b(b + 2) \quad . \quad . \quad (4)$$

(where T′ is the T of the previous paragraph divided by 32·2), corresponding to equation (6), page 25.

$$\text{Put } B = (1 + b)^2 - 1 = b(b + 2) \quad . \quad . \quad (B)$$

Then we have from equations (1), (2), and (4), using equation (3) to eliminate k_L—

$$K' = (BR_1 + R_1 + R_2)\frac{1}{10^4} + \frac{W/V^2}{L/D} \quad . \quad . \quad (5)$$

$$W/V^2 = \cdot 0051\lambda k_{L max}(S + BS') \quad . \quad . \quad . \quad . \quad (6)$$

and

$$B = \frac{10^5 T'}{1\cdot 39 d^2 V^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$\therefore\ W/V^2 = \cdot 0051\lambda k_{L max}S + \frac{510}{1\cdot 39}\frac{\lambda k_{L max}S'T'}{d^2V^2} \text{ from (6) and (7)}$$

$$\therefore\ V^2 = \frac{W - 367\dfrac{\lambda k_{L max}S'T'}{d^2}}{\cdot 0051\lambda k_{L max}S} \quad . \quad . \quad . \quad . \quad (V^2)$$

From equations (7) and (V^2) we have—

$$B = \frac{10^5 T' \times \cdot 0051\lambda k_{L max}S}{1\cdot 39 d^2\left[W - 367\dfrac{\lambda k_{L max}S'T'}{d^2}\right]}$$

$$= \frac{367\lambda k_{L max}ST'}{Wd^2 - 367\lambda k_{L max}S'T'}$$

$$= \frac{aS}{Wd^2 - aS'}$$

where

$$a = 367\lambda k_{L max}T' \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

$$\therefore \text{ from (5)}\quad K' = \frac{aSR_1}{10^4(Wd^2 - aS')} + \frac{R_1 + R_2}{10^4} + \frac{W/V^2}{L/D} \quad . \quad (8)$$

$$\text{Now}\quad V^2 = \frac{Wd^2 - aS'}{\cdot 0051\lambda k_{L max}Sd^2}$$

$$\therefore\ \frac{W/V^2}{L/D} = \frac{\cdot 0051\lambda k_{L max}SWd^2}{L/D(Wd^2 - aS')}$$

$$\therefore\ K' = \frac{aSR_1}{10^4(Wd^2 - aS')} + \frac{R_1 + R_2}{10^4} + \frac{\cdot 0051\lambda k_{L max}SWd^2}{L/D(Wd^2 - aS')} \quad .$$

This equation enables us to find K′ for a range of values of λ and thus, by plotting, to find and use its minimum value for flying

off the deck. Generally, however, the angle of attack for the run will be determined by practical considerations, but as near as may be to the angle of least resistance of the wing.

In this case, find the value of λ corresponding to this angle on model tests (this is quite near enough) and then find K′ from the above equation.

For landing, we may take $\lambda = 1\cdot0$ and $T' = 0$, $\therefore a = 0$ and we have—

$$K' = \frac{R_1 + R_2}{10^4} + \frac{\cdot0051 k_{L max} S}{L/D}$$

where L/D is the value corresponding to $\lambda = 1\cdot0$.

In the case where the machine is designed with the propeller axis at an unusual angle, little error will follow from using the same value of K′ as is found above. It would be very laborious to carry the approximation any further, but if desired a method corresponding to that used in the following investigation may be employed.

K′ is in engineer's units and we have to find K, which is in dynamical units. This is done by the equation—

$$K = 14\cdot95K'.$$

Minimum Flying Speed when Getting Off.—We are dealing only with the case where $\lambda = 1\cdot0$, *i.e.* the machine is flying at its stalling angle. We will work in the engineer's units used generally in other chapters than this.

Consider a side view drawing of the machine, so set that the wings are at the stalling angle of incidence: to do this we take the stalling angle of incidence of the model tests as being sufficiently accurate.

Then the wind speed, V miles per hour (of which an estimated stalling value must be taken at this stage) is horizontal.

Now we know the thrust in pounds T′, and we can write down the equation

$$B = \frac{10^5 T'}{1\cdot39 d^2 V^2} \quad . \quad . \quad . \quad . \quad (1)$$

where $B = (1 + b)^2 - 1$, as on page 68.

$$\therefore b = \sqrt{B + 1} - 1 \quad . \quad . \quad . \quad (2)$$

Now consider the direction taken by the slip stream on leaving the propeller: it has a component V in the direction of the relative wind (horizontal) and a component bV parallel to the propeller shaft.

Now combine these two components graphically on the side-view drawing mentioned above, so as to get the direction of the slip stream. Draw a line in this direction through the centre of the propeller boss to cut the line joining the leading edges of the machine in the side-view drawing at a point A.

Now turn to the front-view drawing of the machine and mark in the point A.

With A as centre describe a circle of d inches diameter and *use this circle instead of the propeller circle to compute S′.*

Turn again to the side-view drawing and note the angle of incidence of the wings relative to the resultant slip stream direction. Then look up the corresponding lift coefficient $k_L{}'$ for this angle of attack from the model tests.

$$\text{Now W} = \cdot 00237[k_{L\,max}(S - S') + k_L{}'(1 + b)^2 S'](1 \cdot 467V)^2$$

$$\therefore V = \sqrt{\frac{196W}{k_{L\,max}(S - S') + k_L{}'(1 + b)^2 S'}}.$$

V, of course, is in miles per hour. Therefore the value of v_m in feet per second is given by the equation

$$v_m = 1 \cdot 467V.$$

CHAPTER VII.

WATER PERFORMANCE.

General.—The seaplane or flying boat is at an advantage relative to her sister of the land in that the sea is an aerodrome that is always large enough either for run to get off or for run after alighting: there is, therefore, no need to calculate these lengths.

Again, when a seaplane is launched from the deck of a ship she is fitted with a wheel chassis for the purpose, and therefore in this case reference can be made to the work of the previous chapter, pages 61 and 64.

On the other hand, while it is true that any aeroplane that can fly can get off the ground, it is *not* true that any seaplane that can fly can get off the water. We must therefore investigate, for a seaplane or flying boat, the criterion that she can get off in a calm—getting off head to a wind need not be considered as it is an easier condition than getting off in a calm.

Model Tests.—Let L be any fixed linear dimension (say the overall length) of a flying boat hull, or of the complete float system of a seaplane, and let l be the corresponding dimension of a geometrically similar small model of it.

Let W_w be the portion of the weight of the seaplane that is water-borne and let w_w be the corresponding quantity for the model.

Let V be the speed of the seaplane at the time in question and let v be that of the model.

Further, let the model be run at the same angle as the full scale machine.

Let F be the resistance due to the water for the full scale machine and f that for the model.

Then

$$\frac{F}{V^2L^2} = \frac{f}{v^2l^2} \quad . \quad . \quad . \quad . \quad (1)$$

provided that

$$\frac{V^2}{L} = \frac{v^2}{l} \quad . \quad . \quad . \quad . \quad (2)$$

and

$$\frac{W_w}{L^3} = \frac{w_w}{l^3} \quad . \quad . \quad . \quad . \quad (3)$$

This is Froude's law of similarity, on which all model testing of hulls and floats for flying boats and seaplanes depends.

When a model is tested specially for a design of seaplane, the model is made to a convenient scale and the relation of W_w to V is worked out from the consideration that W_w plus W_a (the air-borne weight) equals W (the total weight) while

$$\frac{W_a}{V^2} = \frac{W}{V_1^2}$$

where V_1 is the minimum flying speed of the machine at the angle of the wings corresponding to the angle at which the float is to be run.

The model is then run at a series of speeds from quite slow ones up to v_1', loaded at each speed to the value of w_w which satisfies the above, and then f is measured.

We have, then, a series of values of f for a series of values of v.

Then $$F = \frac{L^2}{l^2}\frac{V^2}{v^2}f \quad \text{from (1)}$$

$$= \frac{L^2}{l^2}\frac{L}{l}f \quad \text{from (2)}$$

$$\therefore F = \left(\frac{L}{l}\right)^3 f \quad . \quad . \quad . \quad . \quad . \quad (F)$$

and $$V = \sqrt{\frac{L}{l}}v \quad . \quad . \quad . \quad . \quad . \quad (V)$$

which is all that we require from the model test to enable us to calculate whether the machine will get off in still air.

If the model is made specially for the machine in question, the designer will probably be given F plotted on V instead of f plotted on v, and therefore he will not have to do any converting.

If, on the other hand, he works—as is more usual—by copying the underwater lines of some float system of which tests are available, then he must convert the data available to his own case by means of equations (F) and (V), being careful to see that the conditions on page 71 are all satisfied. This method can be used equally to convert from model figures to the machine in question or from any available set of full scale figures to the

machine in question: in the latter case, F and V refer to the designer's machine, while f and v refer to the full scale figures he is working from.

Note.—Float tests are usually quoted in knots instead of in miles per hour. Such tests should be converted to miles per hour for facility in working in with other performance calculations. If a speed **V** knots is given, this is the same as a speed V miles per hour where

$$V = 1{\cdot}151\ \mathbf{V}.$$

Water Performance.—Now let V_1 miles per hour be the getting off speed, *i.e.* the minimum flying speed for the angle of attack of the wings when running on the water, and let P_1 be the horse-power corresponding to V_1 on the machine performance curve. Let V miles per hour be any speed less than V_1. Then the value of F, the water resistance in pounds, for the speed V is known.

Now the total machine resistance when flying at speed V_1 is

$$\frac{375P_1}{V_1} \text{ pounds.}$$

Therefore at speed V on the water, the total air resistance is

$$\frac{375P_1}{V_1}\left(\frac{V}{V_1}\right)^2 \text{ pounds.}$$

Also the water resistance is F pounds, therefore the gross resistance is $\frac{375P_1}{V_1}\left(\frac{V}{V_1}\right)^2 + F$ pounds.

Now let P_w be the total effective horse-power required to maintain the speed V on the water, then

$$P_w = \frac{V}{375}\left[\frac{375P_1}{V_1}\left(\frac{V}{V_1}\right)^2 + F\right]$$

$$\therefore P_w = P_1\left(\frac{V}{V_1}\right)^3 + \frac{VF}{375} \quad . \quad . \quad . \quad . \quad (P_w)$$

With the aid of equation (P_w) we can now plot P_w on a base of V from $V = 0$ to $V = V_1$ (where the curve runs into the machine performance curve).

Now let the constant torque propeller curve, *i.e.* the curve of P_T on V, be plotted on the same paper.

The Getting Off Condition.—The criterion that the machine

should succeed in getting off the water in a calm is simply that the P_T curve has to lie above the P_w curve all along.

It will always be found that there is a pronounced hump on the P_w curve, which may cut into the P_T curve: if this happens the machine will not get off in a calm. In fact, it will be found that a seaplane which has enough power to get off in a calm always has quite a respectable margin of power for climbing, once it is in the air.

PART II.

PRACTICAL PROCEDURE.

CHAPTER VIII.

BODY RESISTANCE.

General.—The object of the work is to find the numerical values of R_1 and R_2, that is to say, the aggregate resistance in pounds at 100 miles per hour in air of standard density of parts of the machine subject and not subject to the action of the slip stream respectively.* If great accuracy is not sought for, the slip stream effect will be disregarded and the total figure will then be R.

The procedure is simple and not particularly laborious: it consists in making two lists, one of parts in the slip stream, the other of parts outside it, and noting the necessary areas and other dimensions the need for which will appear in the following pages. Then with the aid of the following pages a figure r, the resistance of the item in pounds at 100 miles per hour, is entered opposite each item of the list and finally these figures are added up and give R_1 and R_2.

It is most advisable to deal with the machine as a whole and not to attempt to work to half the machine and double the result.

Of course at the present stage the diameter of the propeller is not known, and therefore the propeller circle is not accurately known. Therefore we must either make a reasonable guess at the propeller diameter and leave it at that; or we can disregard slip stream, find R and then go far enough with the work of the next two chapters to determine a propeller diameter, and finally return and repeat the work of this chapter; alternatively we can make a reasonable guess at the propeller diameter, find R_1 and R_2, proceed with the work of the next two chapters till we find the propeller diameter, and then come back and correct. See also the method used on page 182.

* Bodies in front of a propeller are to be taken as not subject to slip stream action: further, it is sufficiently accurate to assume that the parts affected by slip stream action are those within the propeller circle in the front-view drawing of the machine; the slip stream of a propeller contracts, but this is offset by the forcible separation due to the presence of the body: also the slip stream is deflected downwards by the action of the main planes, but in this respect "what is lost on the swings is gained on the roundabouts".

None of these processes is really satisfactory, but they are the best we can do.

Large Bodies.—First obtain the maximum cross-sectional area of each such body in square feet, from the front-view general arrangement drawing, for instance, and note all such features as cockpits, windscreens, or projecting engine cylinder heads and (in the case of seaplane floats and flying boat hulls) steps: these items are to be dealt with later on in the list: at the moment they are to be neglected.

Let a be the maximum cross-sectional area in *square feet:* then the values of r for large bodies of different types are given in the following table:—

Type of Body.	Value of r.
Tractor fuselage	$2{\cdot}5a$
Pusher nacelle	$7a$
Engine egg	$7a$
Flying boat hull	$1{\cdot}3a$
Pontoon float	$6a$

Tail Unit.—The external structural members, king levers, etc., are to appear in another part of the list under their proper headings of struts, wires, etc.: for the present it is only a question of finding the values of r for the fabric-covered surfaces themselves. First obtain the area in *square feet* of the tail plane (including elevators) and the fin (including rudder), from the plan and side view general arrangement drawings, for instance.

Then the following table gives the required values of r, a being in each case the appropriate area in *square feet:*—

Surface.	Value of r.
Tail plane and elevators	$\cdot 78a$
Fin and rudder	$\cdot 58a$

Struts.—First the various struts used on the machine must be collected into groups, in each of which all members have their cross sections geometrically similar. Then the aggregate frontal area of each group is to be noted, and for this purpose it is convenient to use *one foot long by one inch wide* as the unit of area: tapered struts are accounted for by taking their actual frontal

area, of course, not the product of their length and maximum width.

Now take the geometrical shape of the cross section (or take the shapes one at a time if more than one type of section is used on the machine) and plot it on tracing paper to a scale which will make it $1'' \times 4''$, reducing the section in different ratios horizontally and vertically if it is not of 4 : 1 fineness ratio: before doing so, however, if the section has a pointed tail this should be rounded as may be judged most suitable, see C, page 80. Next, lay the tracing over shapes A and B, page 80, and then, from a consideration of the shape of the traced section relative to the shapes A and B, particularly at the tail end, and taking account of the actual fineness ratio of the struts on the machine, a suitable value of x can be read off from the curves (or at some interval between the curves, if the strut is of intermediate type) at the top of page 80. It is to be noted that in considering the aspect ratio of inclined struts such as chassis struts and the gap struts of staggered biplanes, the downwind dimension is to be taken and not the dimension at right angles to the length of the strut: also when obtaining the frontal area, the length is taken in front view—not the true length.

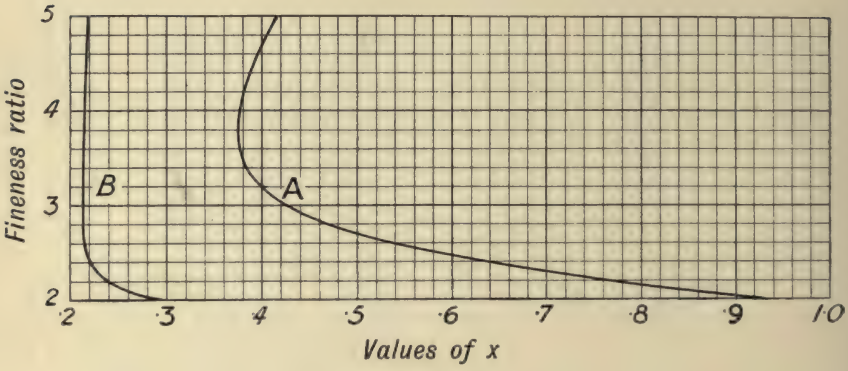
Then if a is the aggregate area of front view in terms of a unit $1'' \times 1' - 0''$, the aggregate value of r for the batch of struts is given by—

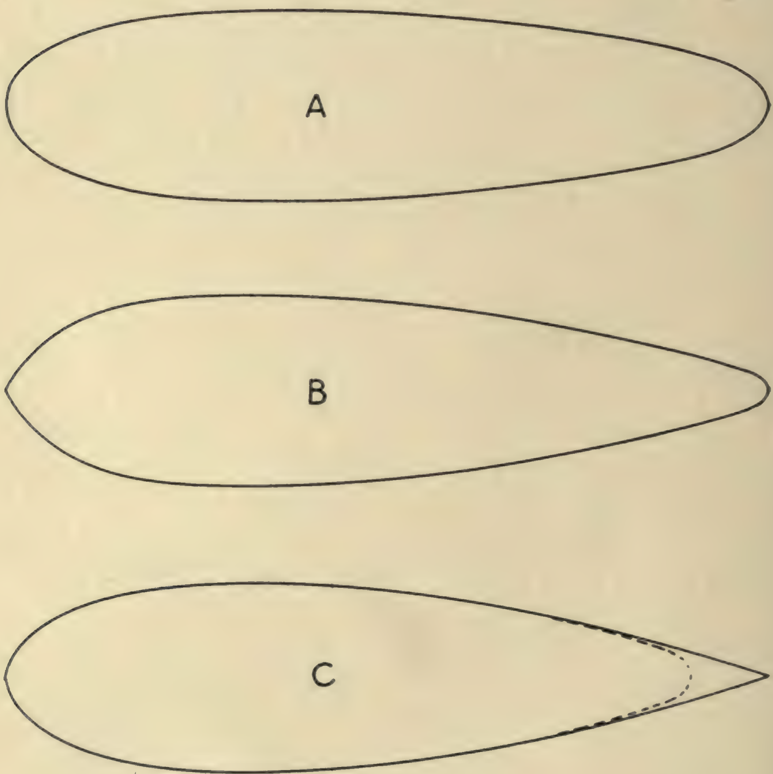
$$r = ax.$$

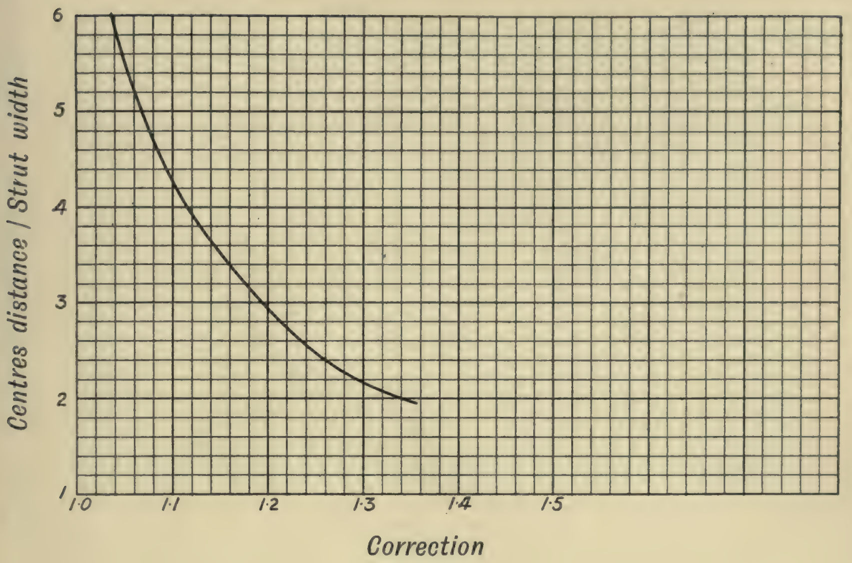
Correction for Shielding.—The shielding effect between front and rear gap struts is inappreciable, and no allowance should be made for it.

Correction for Interference.—If any pairs of struts occur side by side in the machine at a distance apart which is not great compared to the width of either strut, then the resistances of such struts are to be increased by multiplying them by the value of the correction read off the curve at the top of page 81. If the two struts are of different widths, the mean of the two widths should be used in reading the curve.

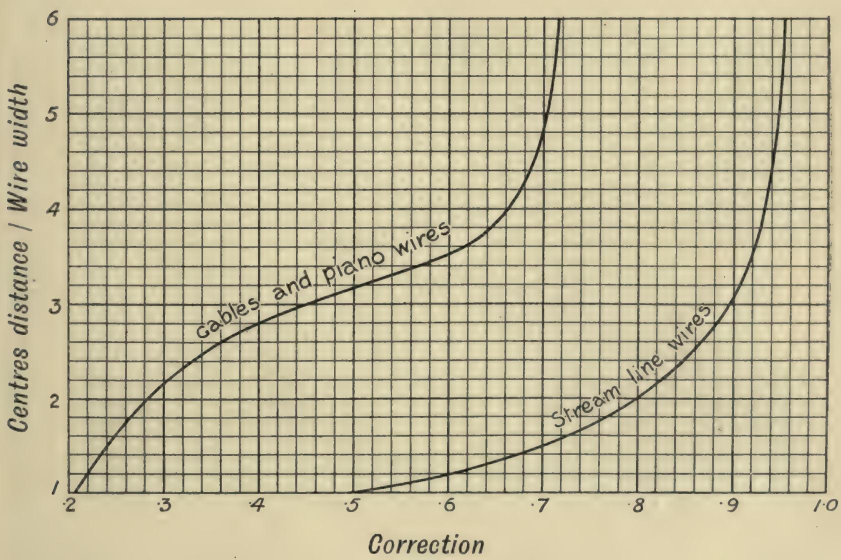
Wires.—First note the overall length l of each wire in *inches;* this may conveniently be done by scaling off a front-view general arrangement drawing of the machine—in fact, the front view length and not the true length is to be taken: also note the diameter d' of the wire in *inches* in the case of piano


Fineness ratio
5
4
3
2
B
A
·2
·3
·4
·5
·6
·7
·8
·9
1·0
Values of x


A
B
C



6
5
4
3
2
1
Centres distance / Strut width
1·0
1·1
1·2
1·3
1·4
1·5
Correction



6
5
4
3
2
1
Centres distance / Wire width
Cables and piano wires
Stream line wires
·2
·3
·4
·5
·6
·7
·8
·9
1·0
Correction

wire, or cable, but in the case of stream-line wires, d' is the diameter of the *thread*. Then the value of r is given in the following table:—

Type of Wire.	Value of r.
Stream-line wire . . .	$\cdot 025 d'(l + 250 d')$
Cable	$\cdot 26\ d'(l + 200 d')$
Piano wire	$\cdot 21\ d'(l + 300 d')$

The above formulæ include a suitable allowance for fork ends, turnbuckles, splices and ferrules, appropriate to each type of wire. It is to be noted that in order that this allowance may work out right, the formula is to be applied separately to each wire: it is not of course correct to add up a batch of wire lengths before applying the formula.

Correction for Shielding.—As has been explained in Chapter I., it is advisable to leave out this correction. If, however, it is preferred to include it in dealing with duplicated wires the procedure is as follows: First obtain in each case the fore and aft dimension of the wires, and the fore and aft centres distance: the latter dimension can be obtained from drawings of fittings. Then read off the appropriate correction from one of the curves at the bottom of page 81: the value of r obtained in the ordinary way is to be multiplied by the correction. The correction is to be applied to *both* the wires.

Wheels.—Let D be the diameter of the wheel, and d' the diameter of the tyre in *millimetres* as ordinarily quoted in defining the size of a wheel, then the values of r corresponding to different methods of fairing the wheel are given in the following table:—

Method of Fairing.	Value of r.
None	$\cdot 000184\ \mathrm{D}d'$
Shields extending to the rim . . .	$\cdot 000113\ \mathrm{D}d'$
Shields extending to the tyre . . .	$\cdot 000062\ \mathrm{D}d'$

Cockpits.—Let l be the maximum width of the cockpit in *inches*, then the value of r is given by the formula—

$$r = \cdot 7 l.$$

It is doubtful whether there is much shielding between two or more cockpits in tandem: no allowance should therefore be made, but each cockpit should be taken at full value.

Windscreens are *not* included in the above.

Radiators.—If a is the total frontal area of the radiator in *square feet*, the resistance is given in the following table:—

Position and Type of Radiator.	Value of r.
Nose radiator with shutters	$14a$
Exposed radiator with shutters	$29a$
Retractable radiator without shutters . . .	$6a$

The above figures apply to standard honeycomb radiators with 10 millimetre by 120 millimetre tubes.

Flat Plates.—Under this heading come such items as windscreens, backs of steps of floats and flying boat hulls, backs of louvres used for cooling engine bays, parts of fittings projecting normal to the wind, and many small miscellaneous parts.

First, all such parts are to be grouped together and their aggregate frontal area a in *square feet* for the complete machine noted, no allowance being made for any shielding which may occur. Then the value of r for these items is given by the formula—

$$r = 29a.$$

Circular Cylinders.—Under this heading are included projecting heads of engine cylinders, wheel axles if unfaired, and sometimes various other odds and ends.

First obtain the aggregate frontal area a in *square feet* of all such parts. Then—

$$r = 31a.$$

Line of Action of Body Resistance.—The height of this line of action is required in certain cases. To find it, first group the items of R in batches corresponding to parts at the same vertical level. Usually these batches will be (1) Gap struts, flying wires, antiflying wires and incidence wires, all reckoned as being at a point half way up the gap; (2) Axle and wheels, reckoned at the level of the wheel centre; (3) Remainder of chassis, reckoned as half way between the wheel centre and the bottom of the fuselage; (4) The fuselage *and all items not included elsewhere*, half way up the fuselage at the centre section

of the machine; (5) The tail plane and elevators, at the height of the centre of the tail plane; (6) The fin and rudder, at the height of their approximate centre of area.

Now let these various batches have resistances in pounds at 100 miles per hour, totalling respectively R_1, R_2, R_3, R_4, R_5, and R_6, and let the heights of the specified points above any convenient datum (such as the top longeron of the fuselage, if this is horizontal, or the line of the propeller thrust, or even a line put in on the drawing for this sole purpose) be a_1, a_2, a_3, a_4, a_5, and a_6.

Then the height of the line of action above the chosen datum is

$$\frac{a_1R_1 + a_2R_2 + a_3R_3 + a_4R_4 + a_5R_5 + a_6R_6}{\mathrm{R}}.$$

The position of the line of action is thus found.

Note that the positive and negative signs of the a's are to be retained during the calculation, of course.

Examples. — For numerical examples illustrating the methods of this Chapter, see Chapter XV., page 147, and Chapter XXII., page 182.

Theory. — For theoretical explanations see Chapter I., page 3.

CHAPTER IX.

WING CHARACTERISTICS.

Definitions.—The lift coefficient in non-dimensional units is denoted by k_L and the drag coefficient by k_D; the lift divided by the drag or k_L/k_D is denoted by L/D; the value of k_L at the stalling angle, *i.e.* the maximum value of k_L attainable, is denoted by k_{Lmax}; k_L/k_{Lmax} is denoted by λ; the centre of pressure coefficient is denoted by k_c.

Aspect ratio is the ratio of the overall span to the length of the main chord of the wing: if the aspect ratio is different on the top and bottom planes, the mean should be taken. Gap/chord is the ratio of the perpendicular distance between the two chord planes to the length of the chord (or to the mean of the top and bottom chords, if these differ). Stagger is the angle in degrees between the line joining the top and bottom leading edges and the normal to the chord. Wing tip shape is considered as the ratio of the span of the rounded part of one tip to the chord. Dimensions means the product of the chord (in inches) of the model used for determining the characteristics of the wing section to be employed and the speed (in feet per second) at which the test was carried out.

Tabular Method.—The corrections on the model test values of k_{Lmax} and L/D are most conveniently handled by means of a table on the following lines * (see next page):—

The left-hand column should be filled in as indicated: the next five columns are then to be filled in by referring to the curves on pages 88 to 97: the curves are marked with numbers corresponding to the λ column, the dotted curve in each case corresponding to k_{Lmax}. Sufficient accuracy is obtained by recording corrections to three decimal places (some of the curves are plotted to scales which render closer reading possible, but this has only been done to prevent the curves being too close together to be seen clearly).

* The device of dividing a table by horizontal lines after every third figure will be found very useful and is recommended for general adoption in manuscript work: it has the advantage of keeping the figures separate without obscuring the page with a complexity of lines.

λ.	Corrections for					Model L/D.	Corrected L/D.
	Aspect Ratio.	Gap/Chord.	Stagger.	Wing Tips.	Dimensions.		
·1							
·2							
·3							
·4							
·5							
·6							
·7							
·8							
·9							
1·0							
k_{Lmax}							

The values of L/D and k_{Lmax} for the model test are then to be filled in in the next column, and finally the last column is to be worked out by multiplying the five corrections and the model figure together horizontally across the page.

Cautions in Applying the Corrections.—When the model wing whose channel tests are being used is *a* 6 : 1 *aspect ratio monoplane with square tips* (as is usually the case), no special caution is necessary. If, however, the test was carried out on a model of any other specification, the model tests must first be corrected back to standard model by *dividing* by the appropriate corrections read off the curves. No long explanation of this point is necessary, as it is quite obvious, but it is perhaps as well to mention it.

Often, nowadays, the designer has at his disposal a model test on a complete set of wings for his machine with correct aspect ratio, gap/chord, stagger, and wing tips. In this case, of course, the only correction to apply to the model tests to get the last column of the above table is the dimension correction.

Presentation of Model Results.—Test results received from a laboratory are usually in the form of tables giving k_L, k_D, and L/D for a range of values of angle of incidence. These can be converted readily with the aid of plotting so as to give the values

of k_c and L/D for values of λ from ·1 to 1·0 as required for use in our table. For convenience a few of the best wings for various types of machine are given hereunder in the required form.*

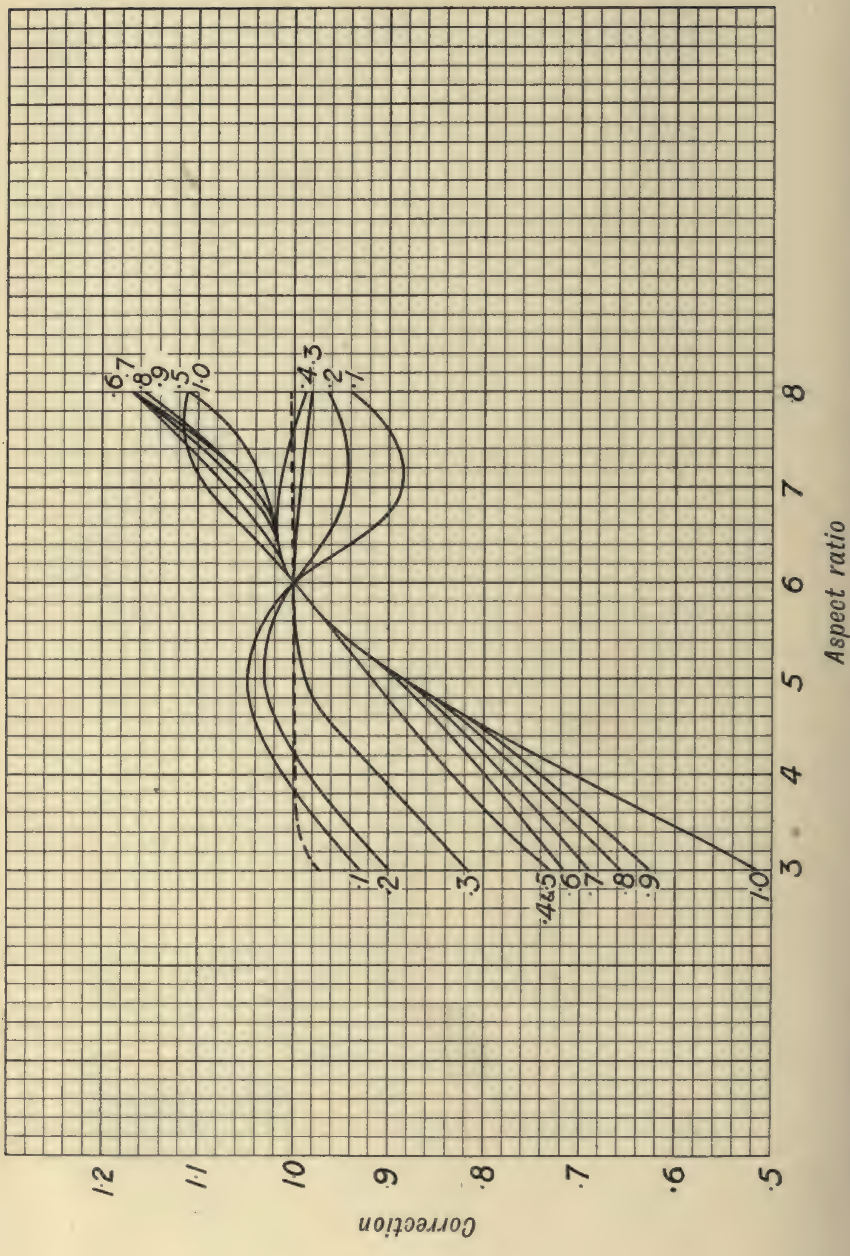
Tests on 6 : 1 Aspect Ratio Monoplane Models with Square Tips. 3 Inch Chord Tested at 40 Feet per Second.

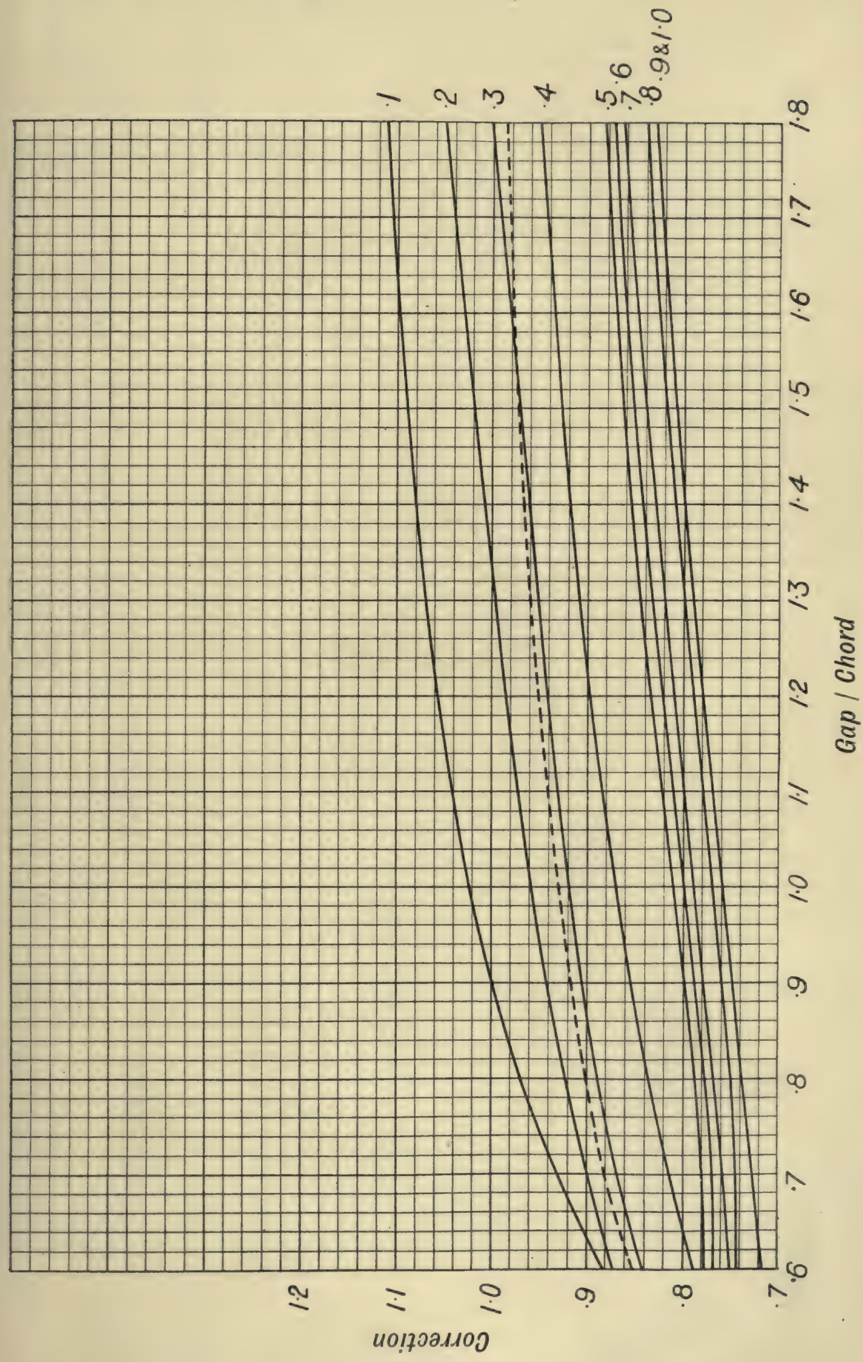
Wing.	R.A.F. 15.		R.A.F. 16.		No. 64.		No. 214.		R.A.F. 19.	
λ.	L/D.	k_c.	L/D.	k_c.	L/D.	k_c.	L/D.	k_c.	L/D.	k_c.
·1	7·00	·463	6·22	·605	5·75	·650	2·51	1·000	2·12	·623
·2	12·59	·373	12·64	·429	11·20	·450	5·58	·645	4·70	·525
·3	16·25	·356	15·63	·384	15·50	·377	9·18	·539	6·98	·485
·4	17·25	·329	16·57	·350	17·80	·341	12·36	·466	8·93	·457
·5	16·75	·310	15·94	·323	17·35	·319	14·50	·415	10·55	·433
·6	15·83	·292	14·86	·307	16·20	·302	14·66	·386	11·73	·415
·7	14·59	·289	13·56	·295	14·80	·291	13·65	·363	11·70	.387
·8	13·11	·285	12·18	·290	13·50	·287	12·70	·348	10·55	·365
·9	11·65	·277	10·82	·284	12·20	·280	11·25	·337	9·73	·352
1·0	7·47	·288	4·77	·289	9·40	·276	8·51	·321	8·30	·341
$k_{L max}$	·513		·568		·617		·728		·845	

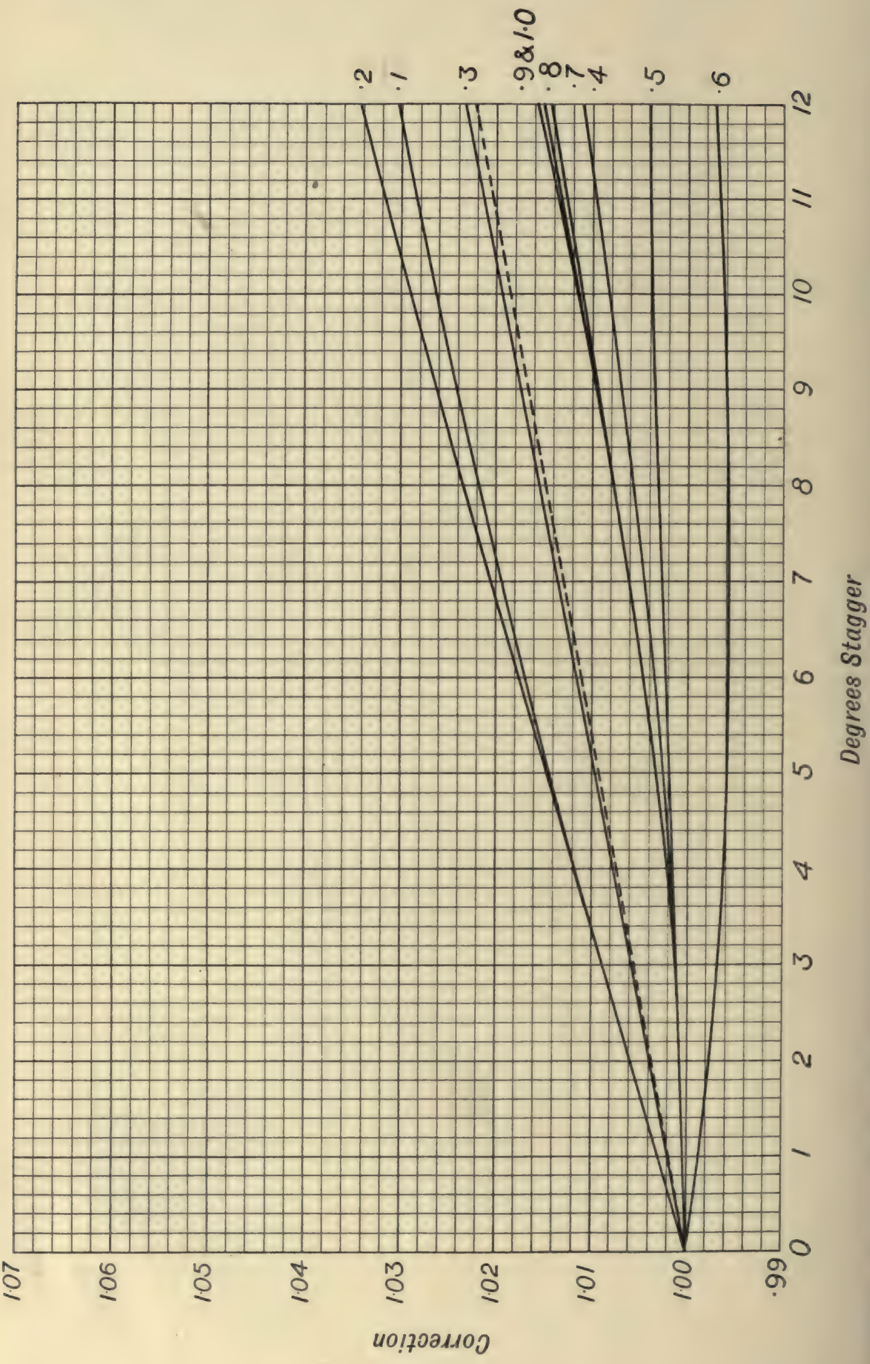
Examples.—For numerical examples illustrating the methods of this chapter, see Chapter XVI., page 152, and Chapter XXII., page 187.

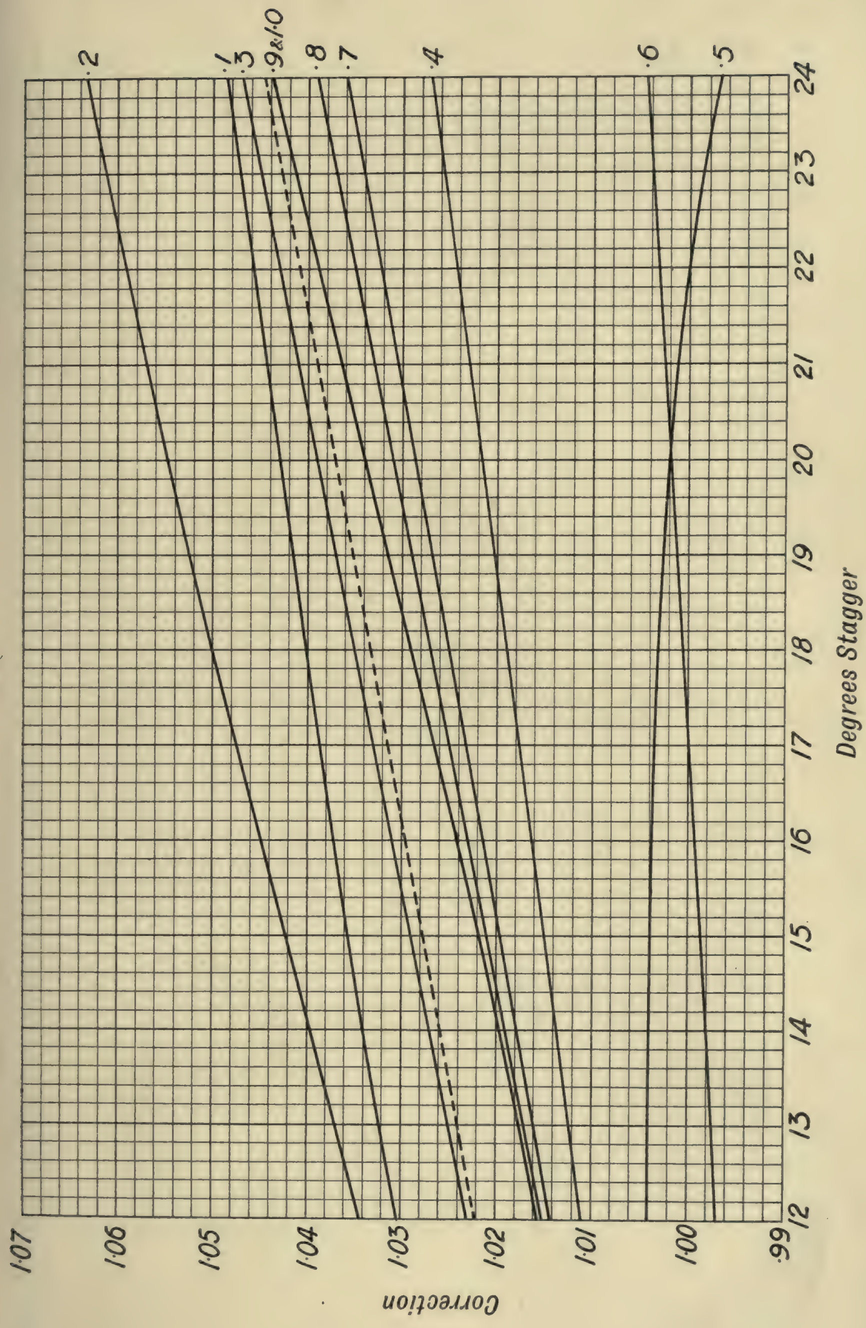
Theory.—For theoretical explanations see Chapter II., page 13.

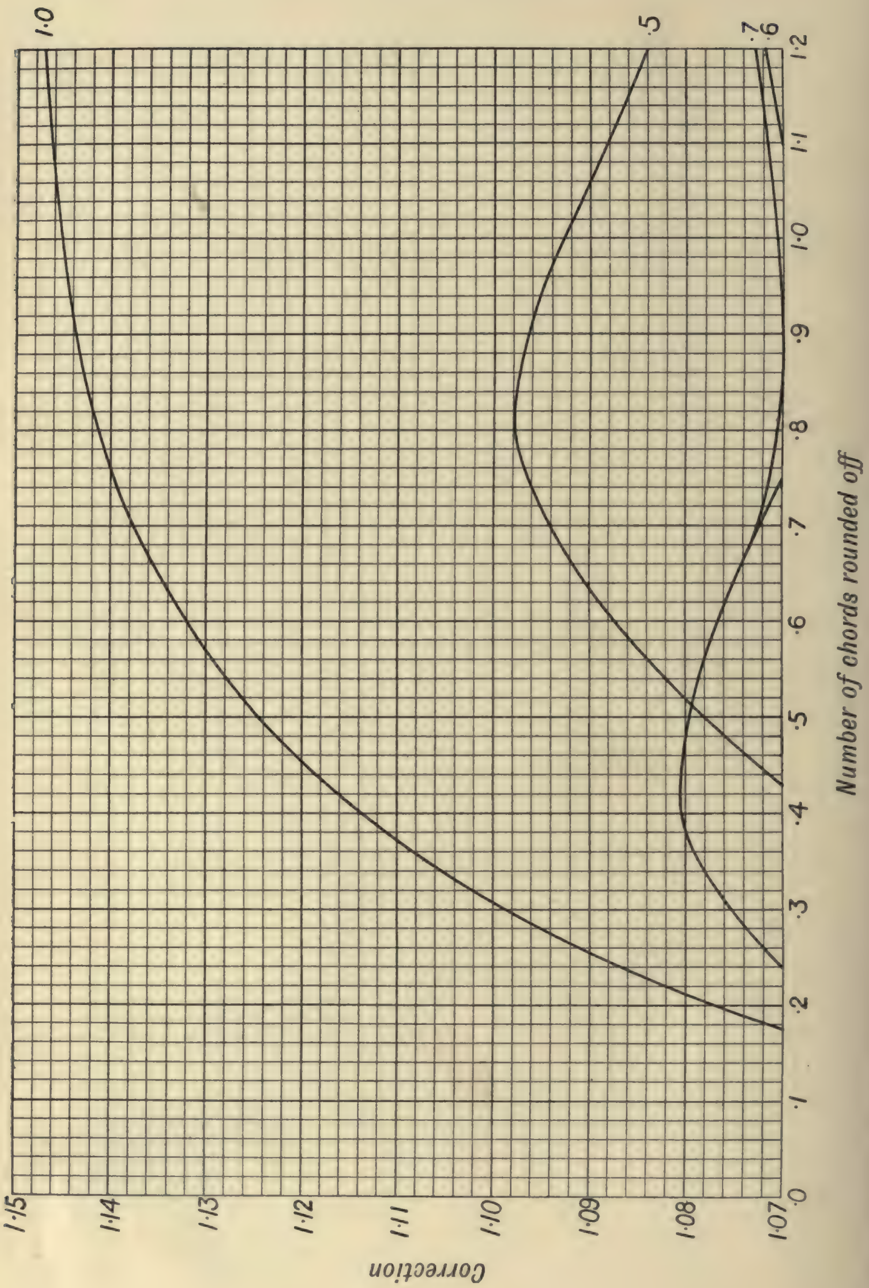
* It is to be noted that a wing which falls entirely inside one of the wings here given when L/D is plotted on k_L is completely inferior to the outer wing. There are many wings which on this test are inferior to the wings here given, but probably few which are better: hence the small number of examples given.

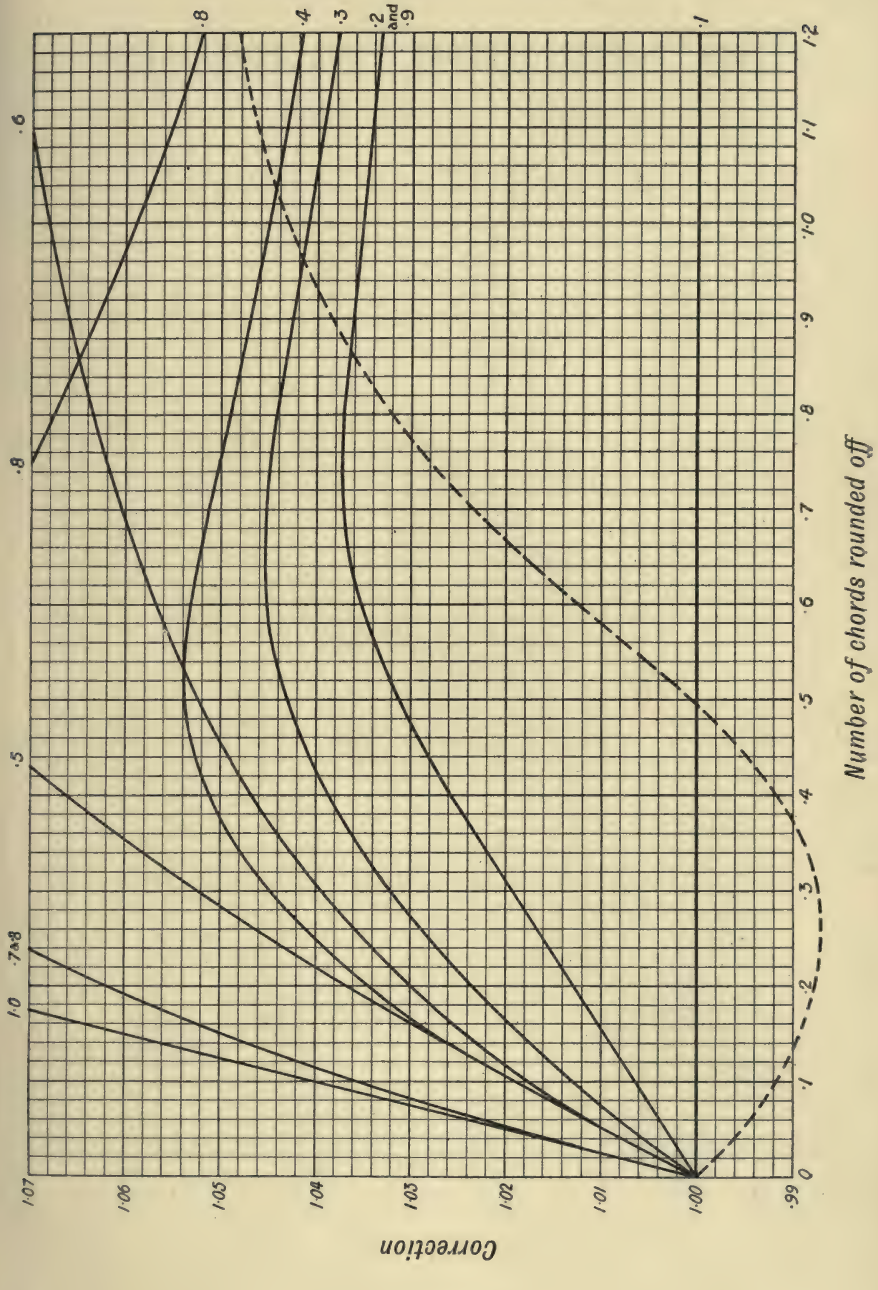

Correction
1.2
1.1
1.0
.9
.8
.7
.6
.5
Aspect ratio
3
4
5
6
7
8
.1
.2
.3
.4&.5
.6
.7
.8
.9
1.0

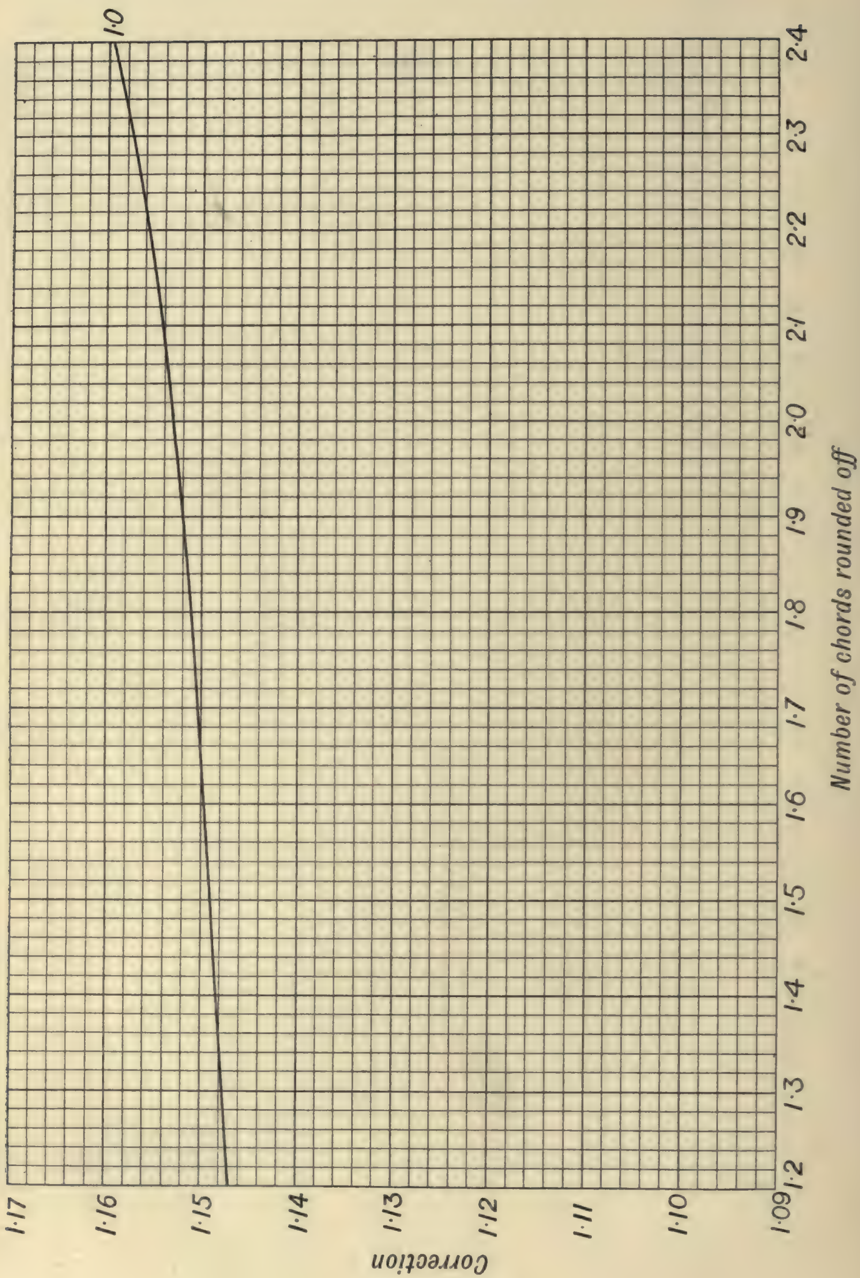

·1
·2
·3
·4
·5
·6
·7
·8
·9 & 1·0
1·8
1·7
1·6
1·5
1·4
1·3
1·2
1·1
1·0
·9
·8
·7
·6
Gap / Chord
1·2
1·1
1·0
·9
·8
·7
Correction

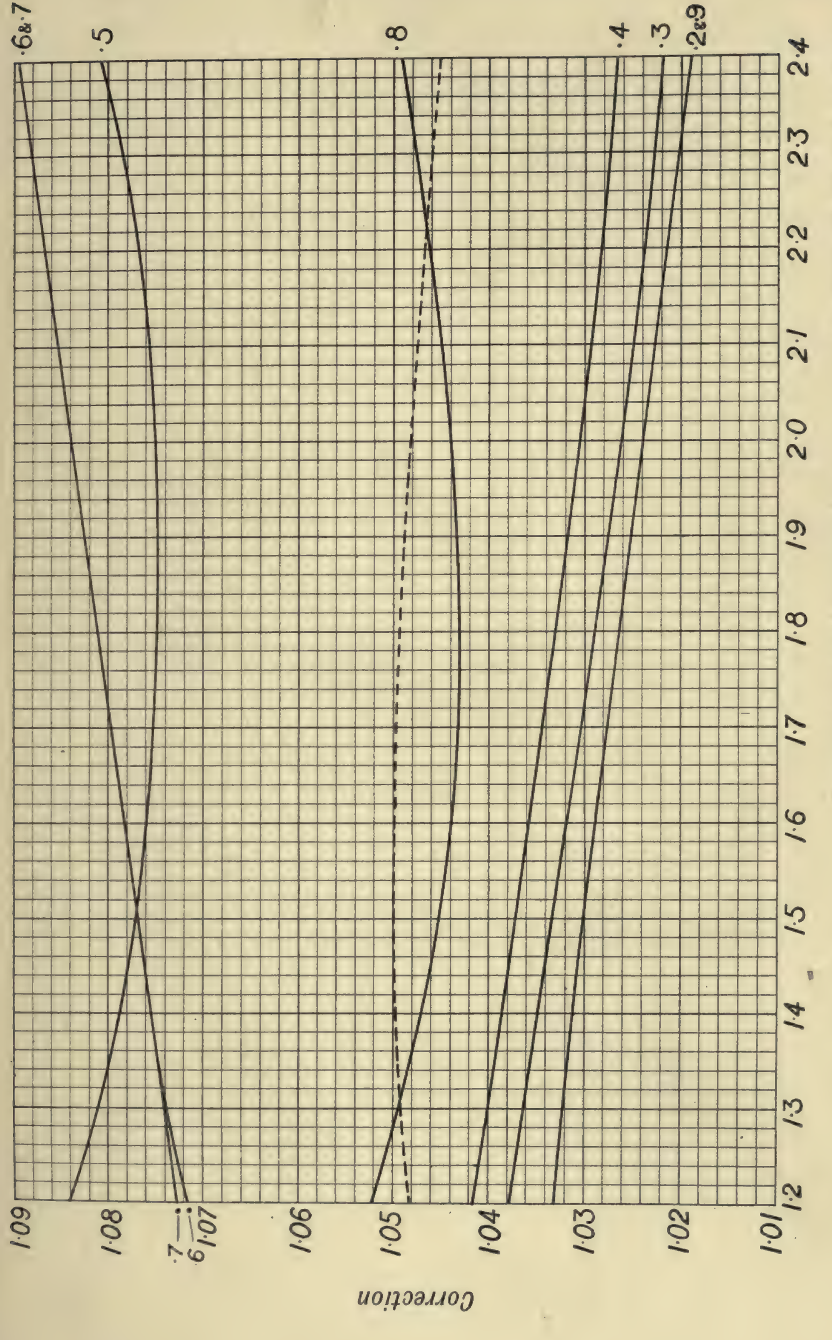

·2
·1
·3
·9&1·0
·8
·7
·4
·5
·6
0
1
2
3
4
5
6
7
8
9
10
11
12
Degrees Stagger
1·07
1·06
1·05
1·04
1·03
1·02
1·01
1·00
·99
Correction


·2
·1
·3
·9&1·0
·8
·7
·4
·6
·5
12
13
14
15
16
17
18
19
20
21
22
23
24
Degrees Stagger
1·07
1·06
1·05
1·04
1·03
1·02
1·01
1·00
·99
Correction

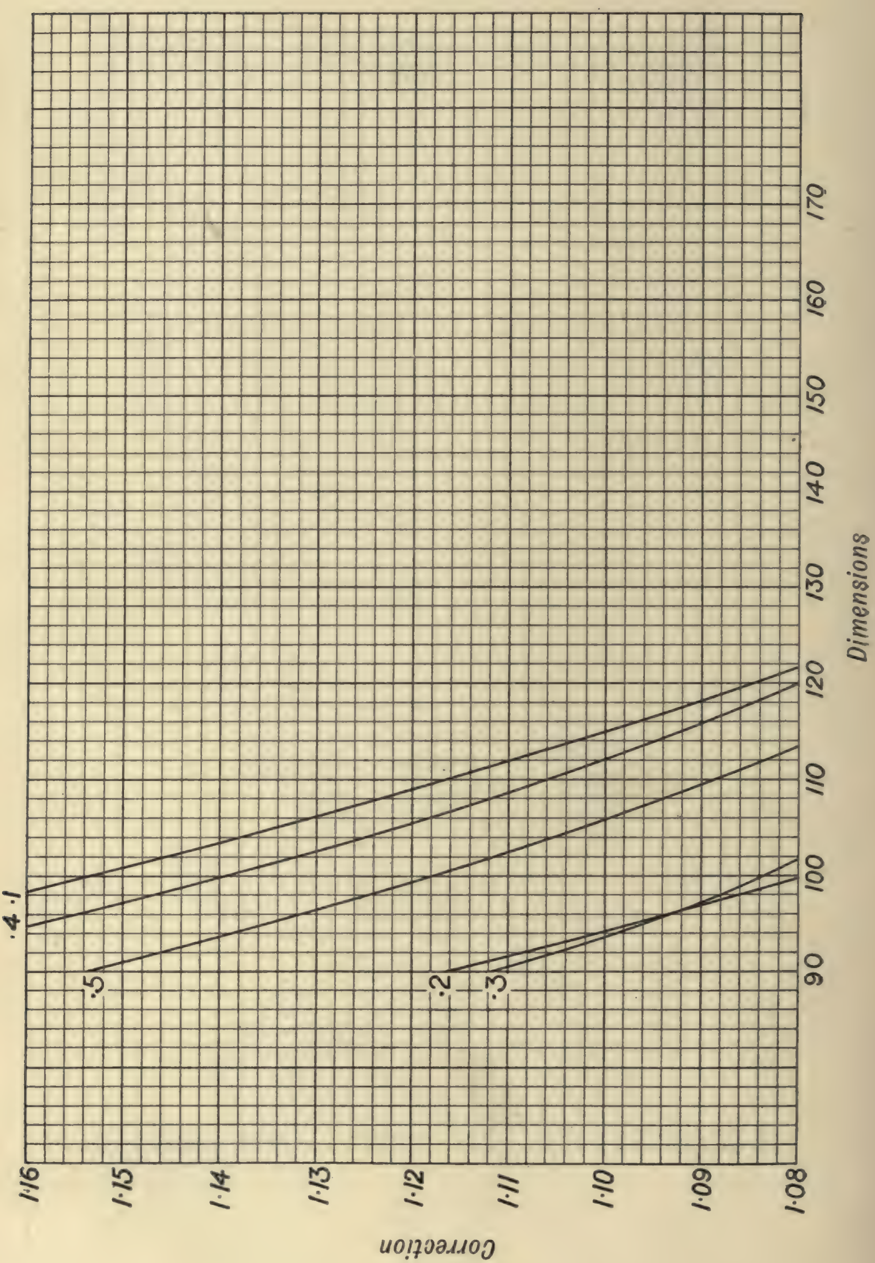

1·0
·5
·7
·6
1·15
1·14
1·13
1·12
1·11
1·10
1·09
1·08
1·07
Correction
0
·1
·2
·3
·4
·5
·6
·7
·8
·9
1·0
1·1
1·2
Number of chords rounded off

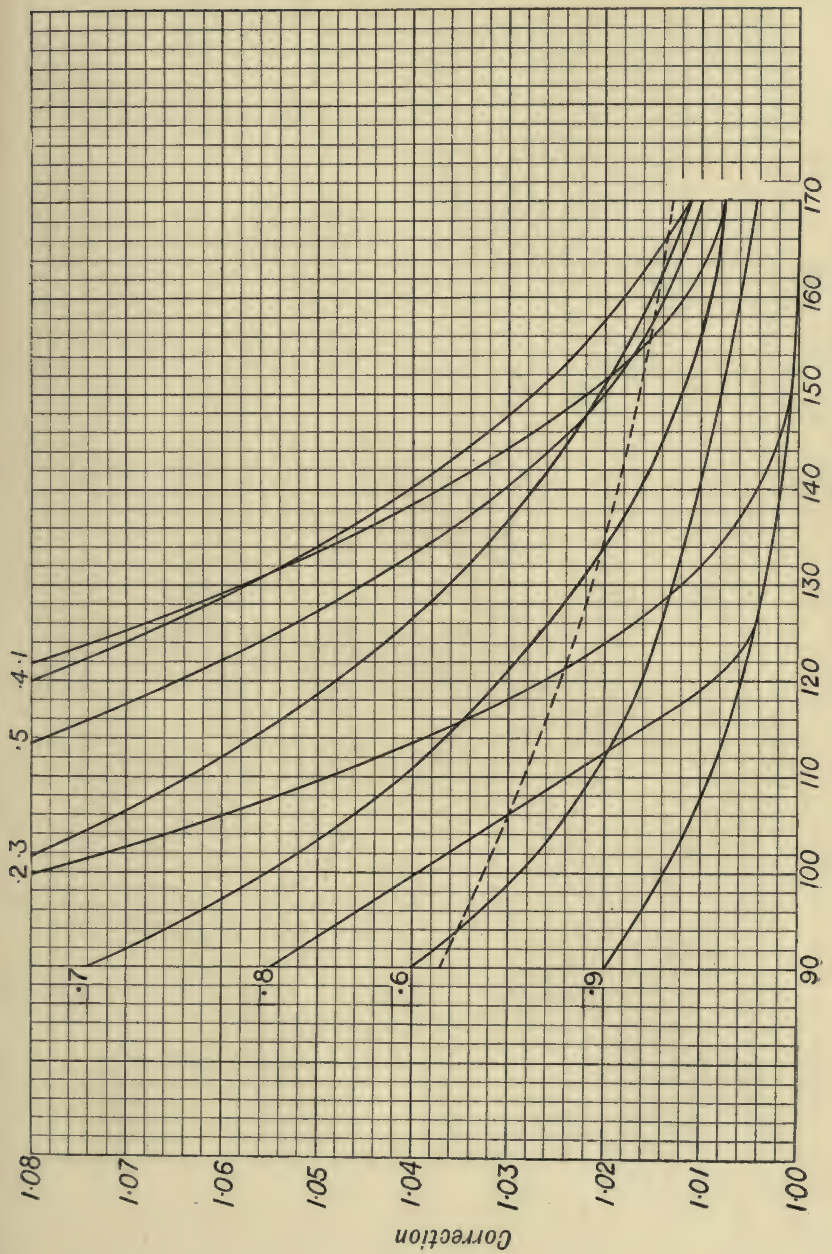

·8
·4
·3
·2
and
·9
·1
·6
·8
·5
·7&8
1·0
1·07
1·06
1·05
1·04
1·03
1·02
1·01
1·00
·99
0
·1
·2
·3
·4
·5
·6
·7
·8
·9
1·0
1·1
1·2
Number of chords rounded off
Correction


1·0
Number of chords rounded off
1·2
1·3
1·4
1·5
1·6
1·7
1·8
1·9
2·0
2·1
2·2
2·3
2·4
Correction
1·17
1·16
1·15
1·14
1·13
1·12
1·11
1·10
1·09



(The correction for λ = ·1 is unity)


Dimensions
90
100
110
120
130
140
150
160
170
Correction
1·16
1·15
1·14
1·13
1·12
1·11
1·10
1·09
1·08
·4 ·1
·5
·2
·3



(The correction for λ = 1·0 is unity)

CHAPTER X.

PROPELLER PERFORMANCE CURVES.

General.—The problem is to find a curve giving P_T, the effective thrust horse-power available for flight, plotted on a base of V, the speed of the machine, on the assumption that the engine is working at normal *torque*, and another curve giving, on a base of V, P_R, the effective thrust horse-power available for flight on the assumption that the engine is working at normal *revolutions*—all in standard density air.

Further, we have to find what these two curves become when the machine is at a given altitude.

Definitions.—

V is the speed of flight in miles per hour in standard density air.

V_0 is the designed speed of the propeller in miles per hour, that is to say, that when the machine is flying (and perhaps climbing) at this speed with the throttle full open, the engine will develop its normal revolutions and its normal B.H.P. in standard density air.

P_T and P_R are the effective thrust horse-power available for flight with the engine working at full throttle and normal revolutions respectively in standard density air.

H is the normal brake horse-power in standard density air.

$$\eta_T = \frac{P_T}{H}.$$

$$\eta_R = \frac{P_R}{H}.$$

n_0 is the normal rate of revolution of the propeller in revolutions per minute.

d is the diameter of the propeller in inches.

$$J = \frac{V_0}{n_0 d}.$$

σ is the ratio of the air density at an altitude to standard air density, and is plotted on page 104.

V′ is the speed of flight in miles per hour at an altitude.

P_T' and P_R' are the values at an altitude of P_T and P_R.

Propeller Diameter.—If the propeller has already been designed the diameter is of course known: otherwise the best diameter for an ordinary case is given by the following formulæ due to H. C. Watts:—

For 2-bladed propellers,

$$d = 10{,}000\sqrt[4]{\frac{\mathrm{H}}{53\cdot5 n_0^2 \mathrm{V}_0}}$$

and for 4-bladed propellers,

$$d = 10{,}000\sqrt[4]{\frac{\mathrm{H}}{111 n_0^2 \mathrm{V}_0}}.$$

The designer is here confronted with the necessity of deciding whether to use a 2-bladed or a 4-bladed propeller: this is a problem of aeroplane design, not of aeroplane performance calculation, and, therefore, we will merely here point out the obvious advantages in reduced height of chassis, increased efficiency and steadier running enjoyed by the 4-bladed type as an offset to its greater cost and its inconvenience for transport —the weights being approximately equal in the two cases.

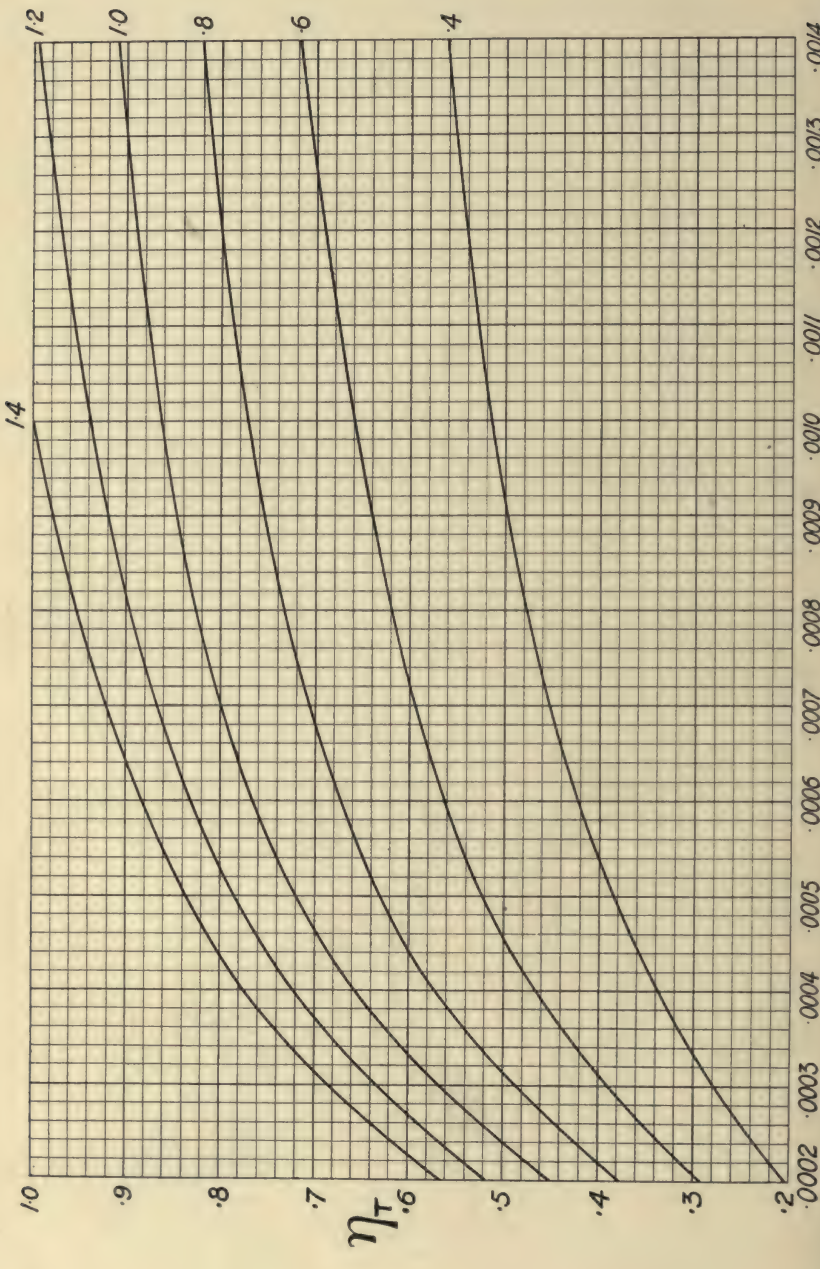
The designer must also now decide what value of V_0 he will use. This again is a design problem, but it is interlocked with the performance of the machine, and hence in a sense with the performance calculation. It may, therefore, be opportune to make some remarks on the choice of V_0.

For a racing machine the end to be aimed at is to make V_0 close to the top speed at ground level—as yet not known. On the other hand, for a weight carrying machine in which top speed is not important, the aim would be to make V_0 approximate more to a speed of good cruising economy. Again, for a machine designed to meet a definite circumscribed specification, if the conditions laid down tend to produce a machine defective in climb, but on the safe side for top speed, a low value of V_0 is suitable, and vice versa.

Probably the best thing to do will always be to make a shot at the best value of V_0, and then, at a later stage, go back and try a larger or smaller value to see if that will improve the machine for its intended purpose: it will be found, by the way, that an alteration of V_0 by 10 miles per hour does not make a lot of difference to the performance.

In brief, the designer is advised to find the best value of V_0 by a system of intelligent trial and error.

The above decisions having been made, d can be found from the appropriate formula, and noted.


1·2
1·0
·8
·6
·4
1·4
·0002
·0003
·0004
·0005
·0006
·0007
·0008
·0009
·0010
·0011
·0012
·0013
·0014
J
1·0
·9
·8
·7
·6
·5
·4
·3
·2
η_T


·6
1·0
·4
1·2
·8
1·0
·9
·8
·7
·6
·5
·4
·3
·2
η_R
·0002
·0003
·0004
·0005
·0006
·0007
·0008
·0009
·0010
·0011
·0012
·0013
·0014
1·4
J

Propeller Performance Curves in Standard Density Air.—Reference must now be made to the curves on pages 100 and 101: these are founded on Bolas' curves for the output of well-designed propellers: they give η_T and η_R plotted on a base of J for a series of values of $\frac{V}{V_0}$, these values being marked on the curves.

The next thing to do is to draw up a table on the following lines:—

$\frac{V}{V_0}$.	η_T.	η_R.	V.	P_T.	P_R.
·4					
·6					
·8					
1·0					
1·2					
1·4					

V_0 has been decided on, d has been found and n_0 is known, therefore J can be found from the formula—

$$J = \frac{V_0}{n_0 d}.$$

The second and third columns in the table can now be filled in by reading η_T and η_R from the appropriate curves at the value of J determined.

V_0 and H being known, the last three columns can be filled up,* remembering that

$$P_T = H\eta_T \text{ and } P_R = H\eta_R.$$

The last three columns contain the necessary data for plotting P_T and P_R on a base of V, *i.e.* the propeller performance curves in standard density air. It is advisable not to plot these curves, however, at the present stage, but to plot them later on over the machine performance curve in standard density air.

Propeller Performance Curves at an Altitude.—First, draw up a table in the following form, allowing for six horizontal lines of figures:—

* It will always occur at $\frac{V}{V_0} = 1·0$ that $\eta_T = \eta_R$ and $P_T = P_R$.

V.	P_T.	V′.	P_T'.

The first thing to do is to fill up the V and P_T columns by copying from the previous table.

The altitude to which V′, P_T', and P_R' refer must now be noted. Probably the altitude will be one at which it is desired to find the top speed of the machine in order to see whether the design meets a particular specification, or perhaps the designer wishes to investigate the performance at a series of altitudes—in which case additional sets of two columns with headings suitably distinguished from one another will be required. However that may be, the altitude is now considered as known.

The altitude being known, σ is found by reference to the curve on page 104.

σ_1 is a quantity defined by the equation—

$$\sigma_1 = \frac{\sigma - p}{q}$$

where $p = \cdot262$ and $q = \cdot738$ for rotary engines, while $p = \cdot161$ and $q = \cdot839$ for stationary engines.

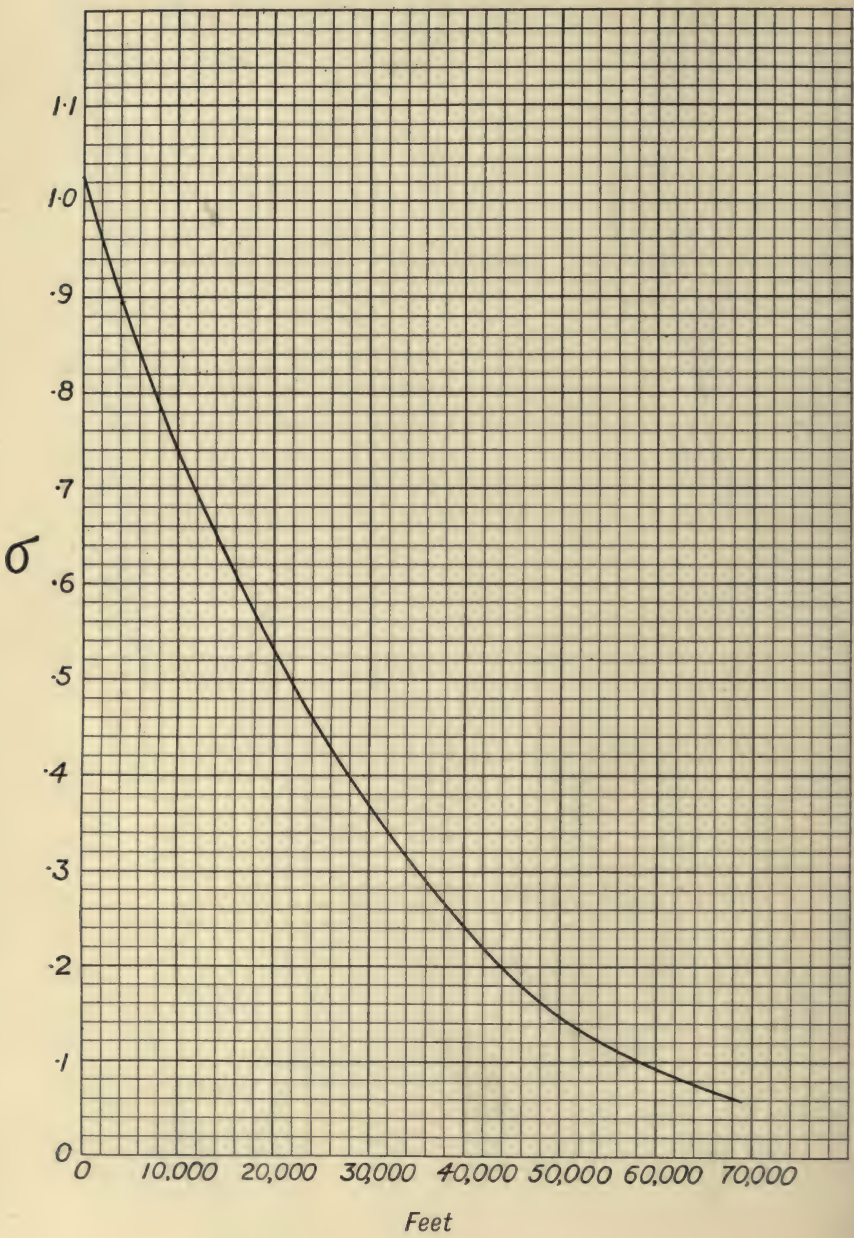
The last two columns of our table can now be filled up by using the formulæ—

$$V' = V\sqrt{\frac{\sigma_1}{\sigma}} \text{ and } P_T' = P_T\sigma_1\sqrt{\frac{\sigma_1}{\sigma}}.$$

The last two columns contain the data for plotting the propeller performance curve for full throttle at an altitude, but again it is preferable to delay the actual plotting until the machine performance curve at the altitude is ready.

It is to be noted, by the way, that the *indicated* torque at the altitude is σ times that for standard density air.

Now draw up a table in the following form, allowing for six horizontal lines of figures :—


1·1
1·0
·9
·8
·7
σ
·6
·5
·4
·3
·2
·1
0
0
10,000
20,000
30,000
40,000
50,000
60,000
70,000
Feet

V.	P_R.	V′.	P_R'.

Then fill in the V and P_R columns by copying from the table of propeller performance in standard density air, and fill in the last two columns by using the formulæ—

$$V' = V \text{ and } P_R' = \sigma P_R.$$

The last two columns contain the data for plotting the constant revolutions propeller performance curve at the altitude. The revolutions to which it corresponds are the same as the full revolutions for standard density air.

The above completes the determination of the propeller performance curves for the general case where the propeller has not yet been designed. In the particular case, however, where the propeller design has already been carried out, the required data will be supplied by the propeller designer, and the above method need not of course be used.

Racing Propellers.—Reference has already been made to the case where it is desired to have a propeller to give the maximum possible top speed to the machine in standard density air: in this case the best value of V_0 is definitely fixed and can be found as follows :—

Having decided the question of 2-bladed or 4-bladed propeller, draw up a table in the following form :—

V_0.	*d*.	J.	η_T.	P_T.

Fill up the first column with three or four values of V_0—say differing successively by 10 miles per hour—so as to form a range of speed ample to cover any top speed that the machine may develop. Then fill in the second column by using whichever of Watts' formulæ applies. Then fill up the third column by using the definition of J. Then fill up the fourth column by

referring to the curves on page 100, making use only of the curve marked 1·0. Finally, fill in the last column by using the formula $P_T = H\eta_T$.

The first and last columns suffice to plot P_T on a velocity base: if this curve is plotted (at a later stage) on the machine performance curve and the speed at which the curves cut is noted, this speed is the best value of V_0 for the racing machine, but this curve of P_T is *not* the propeller performance curve.

If it is desired to get the greatest possible top speed at an altitude instead of in standard density air, the above procedure is not applicable, and it is necessary to resort to trial and error, as will appear later.

Multi-Propeller Machines.—When each propeller is direct driven by a single separate engine, first treat each power unit separately and then add up their separate contributions to P_T, P_R, P_T', and P_R'.

When a propeller is driven by more than one engine or an engine drives more than one propeller, no difficulty will be found in modifying the above processes to meet the case. Note, however, that the mechanical losses in the gearing will modify the values of H, p, and q: this applies also to a geared propeller in the case where part or the whole of the gearing is specially designed for the machine, and has not therefore been included in bench tests at the engine works: in considering this case reference should be made to Chapter III., page 16.

Tandem Propellers.—The case of tandem propellers is too complex to admit of treatment on the lines of the rest of this book. If such a case occurs, reference should be made to "The Design of Screw Propellers for Aircraft," by H. C. Watts.* The subject is treated fully in Chapter XII. of that work.

Examples.—For numerical examples illustrating the methods of this chapter, see Chapter XVII., page 153, and Chapter XXII., pages 188, 195, and 196.

Theory.—For theoretical explanations see Chapter III., page 16.

* Published by Messrs. Longmans, Green & Co.

CHAPTER XI.

MACHINE PERFORMANCE CURVE.

General.—The machine performance curve is the plotting of P on a base of V for standard density air, or of P′ on a base of V′ when flight at an altitude is under consideration.

The Machine Performance Curve in Standard Density Air.—One of the four following methods should be used.

The First Method is for approximate work only.

The Second Method should not generally be used.

The Third Method is for accurate work, except on flying boats.

The Fourth Method is for accurate work on flying boats.

There is also a very useful *Variant of the Third Method* which is recommended for use in the majority of cases.

First Method.—

W is the total weight of the machine in pounds.

S is the total area of the wings in square feet.

R is the total body resistance in pounds at 100 miles per hour, and has been found by the method of Chapter VIII., page 77. If, however, the work of Chapter VIII. has been done in the form of finding R_1 and R_2 instead, then $R = R_1 + R_2$.

$k_{L max}$ and L/D are corrected values for values of λ from ·1 to 1·0 and have already been found by the method of Chapter IX., page 85, where they will be found in the last column of the first table.

V is the speed of the machine in miles per hour in standard density air.

P is the effective horse-power required to maintain horizontal flight at V miles per hour in standard density air.

Now write down the numerical values of W, S, $k_{L max}$, and R.

Then work out and write down the numerical values of—

$$a = \frac{196W}{Sk_{L max}} \text{ and } \beta = \frac{Ra}{10^4}.$$

Then construct a table in the following form :—

λ.	L/D.	$\frac{W}{L/D}$.	$\frac{\beta}{\lambda}$.	$T = \frac{W}{L/D} + \frac{\beta}{\lambda}$.	$V = \sqrt{\frac{\alpha}{\lambda}}$.	$P = \frac{VT}{375}$.
·1						
·2						
·3						
·4						
·5						
·6						
·7						
·8						
·9						
1·0						

Next fill up the λ column as shown and the L/D column as explained in the definition of L/D given above. Then work out the columns in succession.

The last two columns then give the values of V and P, which can be used to plot the machine performance curve for standard density air under the assumptions of the First Method.

Second Method.—

W is the total weight of the machine in pounds.

S is the total area of the wings in square feet.

c is the chord of the wing in feet.

l is the horizontal distance in feet from the c.g. of the machine to the c.p. of the tail plane.*

l' is the horizontal distance in feet from the leading edge of the equivalent chord † to the c.p. of the tail plane.

R is the total body resistance in pounds at 100 miles per hour, and has been found by the method of Chapter VIII., page 77. If, however, the work of Chapter VIII. has been done in the form of finding R_1 and R_2 instead, then $R = R_1 + R_2$.

$k_{L max}$ and L/D are corrected values and have already been found by the method of Chapter IX., page 85, where they will be found in the last column of the first table.

* It is sufficiently accurate for our purpose to assume that the c.p. of the tail plane is at the front tail plane spar.

† The leading edge of the equivalent chord is on the line joining the top and bottom leading edges of the biplane, and at a distance above the leading edge of the bottom plane of ·55 of the gap.

k_c is the centre of pressure coefficient, to which no corrections need be applied.

V is the speed of the machine in miles per hour in standard density air.

P is the effective horse-power required to maintain horizontal flight at V miles per hour in standard density air.

Now write down the numerical values of W, S, $k_{L max}$, R, c, l, and l'.

Then work out and write down the numerical values of—

$$\alpha = \frac{196W}{Sk_{L max}}, \qquad \beta = \frac{R\alpha}{10^4},$$

$$\gamma = \frac{l}{c} \quad \text{and} \quad \delta = \frac{l'}{c}.$$

Then construct a table in the following form :—

λ.	L/D.	k_c.	$\delta - k_c$.	$L/W = \frac{\gamma}{\delta - k_c}$.	$\frac{W}{L/D}$.	$\frac{\beta}{\lambda}$.	$\epsilon = \frac{W}{L/D} + \frac{\beta}{\lambda}$.	$V = \sqrt{\frac{\alpha L/W}{\lambda}}$.	$P = \frac{\epsilon V L/W}{375}$.
·1 ·2 ·3									
·4 ·5 ·6									
·7 ·8 ·9									
1·0									

Next fill up the λ column as shown and the L/D and k_c columns as explained in the definitions of L/D and k_c given above. Then work out the columns in succession.

The last two columns then give the values of V and P, which can be used to plot the machine performance curve for standard density air under the assumptions of the Second Method.

Third Method.—

W is the total weight of the machine in pounds.

S is the *total* area of the wings in square feet (including S′).

S′ is the area in square feet of that part of the wings which

is subject to the action of the slip stream. S′ is obtained by reducing S in the ratio of the sum of the lengths of such parts of the top and bottom leading edges as fall within the propeller circle in front view to the sum of the lengths of the top and bottom leading edges for the whole machine in front view.

c is the chord of the wing in feet.

l is the horizontal distance in feet from the c.g. of the machine to the c.p. of the tail plane.*

l' is the horizontal distance in feet from the leading edge of the equivalent chord † to the c.p. of the tail plane.

d is the diameter of the propeller in inches. A reasonable trial value (if not the final value) of this has already been found by the method of Chapter X., page 98: this value is to be used for the present calculation.

R_1 and R_2 are the body resistances in pounds at 100 miles per hour already found by the method of Chapter VIII., page 77, R_1 corresponding to parts in the slip stream and R_2 corresponding to the rest of the machine.

$k_{L max}$ and L/D are corrected values for values of λ from ·1 to 1·0 and have already been found by the method of Chapter IX., page 85, where they will be found in the last column of the first table.

k_c is the centre of pressure coefficient, to which no corrections need be applied.

V is the speed of the machine in miles per hour in standard density air.

P is the effective horse-power required to maintain horizontal flight at V miles per hour in standard density air.

Now write down the numerical values of W, S, S′, $k_{L max}$, R_1, R_2, c, l, l', and d.

Then work out and write down the numerical values of—

$$a' = 1 - \frac{7{\cdot}2R_1}{d^2}, \qquad b' = \frac{R_1 + R_2}{10^4},$$

$$c' = \frac{367 k_{L max} S'}{d^2}, \qquad d' = {\cdot}0051 k_{L max} S,$$

$$a = \frac{W}{a'd' + b'c'}, \quad \gamma = \frac{l}{c} \text{ and } \delta = \frac{l'}{c}.$$

Then construct a table in the following form:—

* It is sufficiently accurate for our purpose to assume that the c.p. of the tail plane is at the front tail plane spar.

† The leading edge of the equivalent chord is on the line joining the top and bottom leading edges of the biplane, and at a distance above the leading edge of the bottom plane of ·55 of the gap.

λ.	L/D.	k_c.	$\delta - k_c$.	$L/W = \frac{\gamma}{\delta - k_c}$.	$\frac{a'}{\lambda}$.	$\frac{c'}{L/D}$.	$\theta = \frac{a'}{\lambda} - \frac{c'}{L/D}$.	$\frac{b'}{\lambda}$.	$\frac{d'}{L/D}$.	$\phi = \frac{b'}{\lambda} + \frac{d'}{L/D}$.	$V = \sqrt{a\theta L/W}$.	$P = \frac{\phi V^3}{375\theta}$.
·1 ·2 ·3												
·4 ·5 ·6												
·7 ·8 ·9												
1·0												

Next fill up the λ column as shown, and the L/D and k_c columns as explained in the definitions of L/D and k_c given above. Then work out the columns in succession.

The last two columns then give the values of V and P, which can be used to plot the machine performance curve for standard density air under the assumptions of the Third Method.

There is a *Variant of the Third Method*, which is much to be recommended for regular use. It takes account of slip stream effect, but disregards centre of pressure.

To use the variant, simply drop out of the work l, l', c, k_c, γ, and δ, and put L/W = 1.

Then in the table of page 111, the third, fourth, and fifth columns are not required, and L/W can be omitted from the next to the last column.

Fourth Method.—

W is the total weight of the machine in pounds.

S is the *total* area of the wings in square feet (including S').

S' is the area in square feet of that part of the wings which is subject to the action of the slip stream. S' is obtained by reducing S in the ratio of the sum of the lengths of such parts of the top and bottom leading edges as fall within the propeller circle in front view to the sum of the lengths of the top and bottom leading edges for the whole machine in front view.

c is the chord of the wing in feet.

l is the horizontal distance in feet from the c.g. of the machine to the c.p. of the tail plane.*

l' is the horizontal distance in feet from the leading edge of the equivalent chord † to the c.p. of the tail plane.

d is the diameter of the propeller in inches. A reasonable trial value (if not the final value) of this has already been found

* It is sufficiently accurate for our purpose to assume that the c.p. of the tail plane is at the front tail plane spar.

† The leading edge of the equivalent chord is on the line joining the top and bottom leading edges of the biplane, and at a distance above the leading edge of the bottom plane of ·55 of the gap.

by the method of Chapter X., page 98: this value is to be used for the present calculation.

h_1 is the height in feet of the centre line of the propeller shaft above the line of action of the body resistance. This line of action has already been found by the method of Chapter VIII., page 77.

h_2 is the height in feet of the centre line of the propeller shaft above a point half way between the top and bottom leading edges.

R_1 and R_2 are the body resistances in pounds at 100 miles per hour, already found by the method of Chapter VIII., page 77, R_1 corresponding to parts in the slip stream, and R_2 corresponding to the rest of the machine.

$k_{L max}$ and L/D are corrected values for values of λ from ·1 to 1·0, and have already been found by the method of Chapter IX., page 85, where they will be found in the last column of the first table.

k_c is the centre of pressure coefficient, to which no corrections need be applied.

V is the speed of the machine in miles per hour in standard density air.

P is the effective horse-power required to maintain horizontal flight at V miles per hour in standard density air.

Now write down the numerical values of W, S, S′, $k_{L max}$, R_1, R_2, c, l, l', d, h_1, and h_2.

Then work out and write down the numerical values of—

$$a' = 1 - \frac{7{\cdot}2R_1}{d^2} \qquad b' = \frac{R_1 + R_2}{10^4}$$

$$c' = \frac{367k_{L max}S'}{d^2} \qquad d' = {\cdot}0051k_{L max}S$$

$$a = Wl \qquad \beta = a'd' + b'c'$$

$$A' = \beta l' \qquad B' = \beta c$$

$$C' = h_1 b' \qquad D' = h_1 d' - (h_1 - h_2)\beta.$$

Then construct a table in the following form:—

λ.	L/D.	$kc.$	$\frac{a'}{\lambda}.$	$\frac{c'}{L/D}.$	$\theta = \frac{a'}{\lambda} - \frac{c'}{L/D}.$	$\frac{b'}{\lambda}.$	$\frac{d'}{L/D}.$	$\phi = \frac{b'}{\lambda} + \frac{d'}{L/D}.$	$B'kc.$	$\frac{C'}{\lambda}.$	$\frac{D'}{L/D}.$	$\gamma = B'kc + \frac{C'}{\lambda} + \frac{D'}{L/D}.$	$\psi = A' - \gamma.$	$V = \sqrt{\frac{a\theta}{\psi}}.$	$P = \frac{\phi V^3}{375\theta}.$
·1 ·2 ·3															
·4 ·5 ·6															
·7 ·8 ·9															
1·0															

Next fill up the λ column as shown and the L/D and k_c columns as explained in the definitions of L/D and k_c given above. Then work out the columns in succession.

The last two columns then give the values of V and P, which can be used to plot the machine performance curve for standard density air under the assumptions of the Fourth Method.

The Machine Performance Curve at an Altitude.—First draw up a table in the following form, allowing for ten horizontal lines of figures:—

V.	P.	V′.	P′.

This table can, of course, be drawn up as an extension of the table used to find the machine performance curve in standard density air, in which case the first two columns are already filled up. If, however, the table is a separate one, the first thing to do is to fill up the first two columns by copying out the last two columns of the previous table.

Now fill up the last two columns with the aid of the formulæ

$$V' = \frac{V}{\sqrt{\sigma}} \text{ and } P' = \frac{P}{\sqrt{\sigma}}$$

where σ is found for the altitude under consideration by reference to the curve on page 104.

The above procedure is the same whichever method has been used to find the machine performance curve for standard density air.

Examples.—For numerical examples illustrating the methods of this Chapter, see Chapter XVIII., page 155, and Chapter XXII., pages 188 and 194.

Theory.—For theoretical explanations see Chapter IV., page 21.

CHAPTER XII.

AIR PERFORMANCE.

I. Gliding Flight.

Landing Speed on Glide.—This has no particular meaning at an altitude, but the figure is included for the sake of completeness. The First Method is less accurate, but not much so, than the Second.

Definitions.—W is the total weight of the machine in pounds.

S is the total wing area in square feet.

k_{Lmax} is the corrected value already found under the heading of wing characteristics.

σ is plotted against altitude on page 104.

l is the distance in feet from the c.g. of the machine to the c.p. of the tail plane (or front spar of the tail plane will do).

l' is the distance in feet from the leading edge of the equivalent chord (·55 up the gap) to the c.p. of the tail plane.

c is the wing chord in feet.

k_c is the uncorrected value of the centre of pressure coefficient, in this case being limited to refer to its value when $\lambda = 1·0$.

First Method.—In standard density air—

$$V = \sqrt{\frac{196W}{Sk_{Lmax}}}$$

and at an altitude *—

$$V' = \sqrt{\frac{196W}{\sigma Sk_{Lmax}}}.$$

Second Method.—In standard density air—

$$V = \sqrt{\frac{196Wl}{Sk_{Lmax}(l' - ck_c)}}$$

* See footnote to page 117.

and at an altitude *—

$$V' = \sqrt{\frac{196Wl}{\sigma S k_{Lmax}(l' - ck_c)}}.$$

Gliding Angle.—

R is the total body resistance as found by the method of Chapter VIII., page 77.

k_{Lmax} and L/D as found by the method of Chapter IX., page 85.

S is the total wing area in square feet.

θ is the gliding angle in still air, *i.e.* the inclination of the gliding path to the horizontal.

Now write down the numerical values of R, k_{Lmax}, and S.

Then work out and write down the numerical value of—

$$a = \frac{R}{51k_{max}S}.$$

Then construct a table in the following form (see next page).

Next fill up the λ column as shown and the L/D column as explained in the definition of L/D given above. Then work out the columns in succession.

The last column gives the tangent of the gliding angle in still air, and the result is independent of altitude, that is to say,

* As a matter of fact this formula is not quite useless, as in hot countries the average air density is below standard: the same applies to England in the summer, and applies strongly to countries like Mexico and South Africa which lie at a great height above sea-level: on the other hand, the accepted figures give the normal air density in England as rather above standard (see curve on page 104). The matter becomes of importance when designing machines for foreign countries or quoting performance of existing types of machines to foreign buyers. The following table, kindly communicated to the author by G. E. Petty, will be found very useful in this connection: it gives what may be called the effective height in feet of the local sea-level at the worst time of year for a number of places. Every feature of the performance of a machine is, of course, affected by these considerations:—

Adelaide .	. 4150	Cape Verd	. 3620	Mexico .	. 4450	Singapore	3520
Aden .	. 4550	Ceylon .	. 3550	Morocco	. 3400	St. Louis	3550
Archangel	. 2550	Constantinople	3550	New Orleans	3550	Stockholm	2500
Auckland	. 3200	Delhi .	. 4900	New York	. 3400	Sydney	. 3950
Bagdad .	. 4520	Durban .	. 3220	Panama	. 3500	Tokio	. 3400
Banana-Congo	3700	England .	. 2350	Pernambuco	. 3490	Tomsk	. 3320
Bombay .	. 4600	Freetown	. 3550	Pekin .	. 3620	Trinidad	. 3500
Brisbane .	. 3900	Iceland .	. 2180	Port Darwin	. 3920	Tripoli	. 4150
Buenos Ayres	. 4100	Khartoum	. 4600	Pretoria	. 3550	Valparaiso	3450
Cairo .	. 4120	Lima .	. 3150	Quebec .	. 2500	Vancouver	3000
Calcutta .	. 4500	Madras .	. 4400	Rio de Janiero	3520	Winnipeg	3500
Canton .	. 3620	Madrid .	. 3800	Salt Lake City	3580	Zanzibar .	3520
Cape Town	. 3200	Melbourne	. 3950	San Francisco	3400		

λ.	L/D.	$\frac{a}{\lambda}$.	$\frac{1}{L/D}$.	$\tan\theta = \frac{a}{\lambda} + \frac{1}{L/D}$.
·1 ·2 ·3				
·4 ·5 ·6				
·7 ·8 ·9				
1·0				

that for a given value of λ the gliding angle at any altitude is the same as in standard density air.

To find the best gliding angle in still air, plot the last column against λ and note the minimum value: this is the tangent of the best gliding angle in still air. Note also the value of λ to which it refers, as from this, by a reference to the machine performance curve of the First Method for standard density air, the best gliding speed can be noted. At an altitude the best gliding angle is the same as that found above, but the machine speed (which is the pilot's guide) is higher at a greater altitude. Fortunately, however, the Air Speed Indicator Reading suffers the same correction, so that the best gliding speed at any altitude is obtained by holding the machine on an Air Speed Indicator Reading equal to the best gliding speed as found above.

Gliding in a Wind.—

v is the wind speed.

ϕ is the gliding angle relative to the ground.

W is the total weight of the machine in pounds.

Tan θ has already been found for each value of λ in the immediately preceding piece of work.

First write down the numerical values of W, R, and a (taking the last two from the immediately preceding work), and decide on a wide series of values of v (say v_1, v_2, v_3, etc.) in standard density air over which to carry the investigation. Note that the range should extend to negative values of v.

Then construct a table in the following form :—

λ.	L/D.	$\tan\theta$.	$\dfrac{\tan\theta}{L/D}$.	$1+\dfrac{\tan\theta}{L/D}$.	$\dfrac{\lambda}{\alpha}\left(1+\dfrac{\tan\theta}{L/D}\right)$.	$\dfrac{\lambda}{\alpha}\left(1+\dfrac{\tan\theta}{L/D}\right)+\tan\theta$.	$V=\sqrt{\dfrac{10^4 W}{R\left[\dfrac{\lambda}{\alpha}\left(1+\dfrac{\tan\theta}{L/D}\right)+\tan\theta\right]}}$.	$\dfrac{v_1}{V}$.	$1-\dfrac{v_1}{V}$.	$\tan\phi_1=\dfrac{\tan\theta}{1-\dfrac{v_1}{V}}$.
·1										
·2										
·3										
·4										
·5										
·6										
·7										
·8										
·9										
1·0										

Next fill up the first three columns by copying from the immediately previous work. Then work out the columns in succession.

The table should, of course, be extended beyond the limits here shown, by adding three additional columns for each of the values v_2, v_3, etc., of v.

Now plot $\tan \phi_1$, $\tan \phi_2$, etc., all on V as a base, and note the minimum value of each curve: these minimum values are the tangents of the best gliding angles relative to the ground in standard density air against head winds of strength v_1, v_2, etc. Let these values be called $\tan \Phi_1$, $\tan \Phi_2$, etc. Also note the values of V at which they occur, and call them V_1, V_2, etc.

Now draw up another table in the following form, allowing for as many horizontal rows of figures as there are values of v included in v_1, v_2, etc. :—

V.	v.	$\tan \Phi$.	v'.	

Next fill up the v column with the values v_1, v_2, etc., the $\tan \Phi$ column with the values $\tan \Phi_1$, $\tan \Phi_2$, etc., just determined, and the V column with the values V_1, V_2, etc., just determined.

Now from the altitude at which the values of v' are required, find σ from the curve of page 104, and then fill in the v' column, using the relation—

$$v' = \frac{v}{\sqrt{\sigma}}.$$

The table can be extended to the right to include any number of altitudes desired.

Thus we have, at any altitude desired, a column of v' against which the V and $\tan \Phi$ columns can be plotted, giving the *Air Speed Indicator Reading* for best gliding angle relative to the ground, and the tangent of the best gliding angle relative to the ground, all plotted against the velocity of the head wind.

These curves, of course, include such information as is given by the immediately preceding piece of work, but in a slightly less approximate form.

II. Full Power Flight.

Top Speed.—Let the curves of machine performance and propeller performance be plotted over one another. Then the top speed is given by the intersection of the machine performance curve with whichever is the lower of the two propeller performance curves.

For this purpose the First Method of calculating a machine performance curve gives only approximate results, and the Third Method should be used instead.

Top Speed at an Altitude.—The procedure is the same as above, but the curves for the altitude in question are to be used instead of those for standard density air.

Racing Machines.—In the case of a machine solely designed for best top speed in standard density air, the intersections of the P_T and P_R curves with the P curve are identical in standard density air, and at altitudes no ambiguity will be found to arise: the ordinary method given above will be found to apply.

Rate of Climb.—There are three methods available, but, as has been explained in Chapter V., page 35, only the First Approximation should ordinarily be used. The Second Approximation is advisable for machines whose rate of climb is exceedingly large, but to obtain accuracy logarithms should be used with this method rather than a slide rule. The Third Approximation is really hardly worth while: it is, however, included here since, if concluded by the use of the method of the Second Approximation, it is the most accurate at present available.

First Approximation. — Referring to the above-mentioned curves for P, P_T, and P_R plotted on V, and calling P_P the value either of P_T or of P_R, whichever is the lower at each value of V considered, and calling the climb in feet per minute C—

$$C = 33{,}000 \frac{P_P - P}{W}$$

where W is (as usual) the weight of the machine in pounds.

There is a value of C corresponding to each value of V, and, therefore, a plotting is necessary to determine the maximum value of C, and the value of V at which it occurs.

The procedure at an altitude is the same, but the curves for P', P_T', and P_R' on V' are used, of course, instead of the curves for P, P_T, and P_R on V.

The method is only approximate, even if the P curve is got from the Fourth Method.

Second Approximation.—This approximation, though considerably more laborious than the first, gives results differing appreciably from it only in the case of machines which are really very strong climbers: its use should therefore be limited to such cases.

Construct a table in the following form, providing for say half a dozen horizontal lines of figures:—

V.	P_p.	P.	P_p+P.	P_p-P.	$\frac{WV}{375(P_p+P)}$.	$\left\{\frac{WV}{375(P_p+P)}\right\}^2$.	$\frac{P_p-P}{P_p+P}$.	$\left\{\frac{WV}{375(P_p+P)}\right\}^2-\frac{P_p-P}{P_p+P}$.	$\sqrt{\left\{\frac{WV}{375(P_p+P)}\right\}^2-\frac{P_p-P}{P_p+P}}$.	$x=\frac{WV}{375(P_p+P)}-\sqrt{\left\{\frac{WV}{375(P_p+P)}\right\}^2-\frac{P_p-P}{P_p+P}}$.	$1+x^2$.	$C=\frac{176Vx}{1+x^2}$.

Fill in the V column with a few values chosen so as probably to include the speed for best climb. Then fill in the P_p and P columns by referring to the propeller and machine performance curves for these values of V. Then work out the columns in succession: for this it may be necessary to use logarithms in order to obtain sufficient accuracy.

Finally, plot C against V to find the maximum value of C, the rate of climb in feet per minute, and the value of V at which it occurs. For work in standard density air, this value of V is the Air Speed Indicator Reading for best climb.

The same procedure exactly applies to an altitude, but in this case the propeller and machine performance curves for the altitude must of course be used instead of those for standard density air, while the value of V′ obtained is the true air speed, not the Air Speed Indicator Reading for best climb.

Third Approximation.—The procedure in this case is really first to obtain a machine performance curve corrected for the full power climbing slip stream and then to apply the First Approximation to the result—or the Second Approximation instead if the machine is a very strong climber indeed.

With the usual notation, write down the numerical values of R_1, S', $k_{1\,max}$, d, and W and then work out and write down the numerical values of—

$$\alpha = \frac{137{,}500 S' k_{1\,max}}{d^2} \text{ and } \beta = \frac{7 \cdot 2 R_1}{d^2}.$$

Then construct a table in the following form (see next page).

Next fill up the λ column as indicated and the L/D column with corrected values taken from the wing characteristic work. Then turn to the calculation of the machine performance curve done by the First Method or the Second Method (the Third Method or the Fourth Method must *not* be used, as part of the slip stream is already allowed for in them) and fill in the V and P columns. Now turn to the propeller performance curves and fill in the P_p column for the values of V in the V column. Then work out the remaining columns in succession.

Finally, plot P_1 on V_1, forming a new machine performance curve which is practically correct for climbing conditions, and then apply to it the method of the First Approximation.

The procedure at an altitude is to repeat the *whole* of this work, starting from the machine performance curve calculated on the First Method or Second Method and the propeller performance curves, all for the altitude in question instead of for standard density air.

λ.	L/D.	V.	P.	P_P.	$L = \frac{\alpha\lambda P_P}{V}$.	$W - L$.	$V_1 = V\sqrt{\frac{W-L}{W}}$.	βP_P.	$\frac{LV}{375 L/D}$.	$\gamma = \beta P_P + \frac{LV}{375 L/D}$.	$P + \gamma$.	$P_1 = (P+\gamma)\left(\frac{V_1}{V}\right)^3$.
·1												
·2												
·3												
·4												
·5												
·6												
·7												
·8												
·9												
1·0												

In consequence, the accurate calculation of climbs at all altitudes is very laborious.

Fortunately, however, the curve giving the rate of climb plotted against altitude is generally quite a fairly straight one, so that it is sufficient to calculate the rate of climb at three altitudes only, namely, one a little below the estimated ceiling; one for standard density air, and one intermediate altitude.

Times to Altitudes.—C′ is the rate of climb in feet per minute at an altitude. Plot C′ against altitude. Then, if the plotting is *not* approximately a straight line, plot $\frac{1}{C'}$ against altitude in feet. The time to any required altitude is then the area under the curve from the altitude of ground level to the altitude in question.

But if the plotting of C′ on a base of altitude *is* approximately a straight line, draw in that straight line which is the closest approximation to the curve, and let it give $C' = c$ at *ground* level and let it give $C' = 0$ at a_1 feet above the *ground.*

Then the time t in minutes up to any height a feet above *ground* (where a is between 0 and a_1) is given by the formula—

$$t = \frac{2{\cdot}303a_1}{c} \log_{10} \left(\frac{a_1}{a_1 - a}\right).$$

Ceiling.—If these are not already available in curve form, plot the machine performance curve (P on a base of speed) and the *constant torque* propeller performance curve (P_T on a base of speed)—both for standard density air.

Then lay over this the celluloid throttling curves * so that the axis of speed of the throttling curves lies on the axis of speed of the P and P_T curves, while the axis of horse-power of the throttling curves lies on the axis of horse-power of the P and P_T curves.

Now consider say four or five of the throttling curves which cut the P curve towards the lower end of its speed range. Let the intersections be at speeds V_1, V_2, etc.

Now let these same individual throttling curves cut the P_T curve in points where the speeds are V_{T1}, V_{T2}, etc., respectively.

Now draw up a table in the following form, allowing for as many horizontal rows of figures as there are points to consider:—

* For the method of making these see page 126.

V.	V_T.	$\sigma_1 = \left(\frac{V}{V_T}\right)^2$.	$q\sigma_1$.	$\sigma = q\sigma_1 + p$.	Altitude.

It will be remembered that p = ·161 and q = ·839 for stationary engines, while p = ·262 and q = ·738 for rotary engines.

Then fill up the first two columns with the values V_1, V_2, etc., and V_{T1}, V_{T2}, etc., work out the next three columns, and then fill up the last column with the aid of the curve of page 104.

Now plot altitude against V and note the maximum value of the altitude (which is the maximum ceiling) and the value of V at which it occurs (which is the Air Speed Indicator Reading at the ceiling).

III. Throttled Flight.

Slowest Flying Speed.—The slowest flying speed is different from the landing speed on glide, already found on page 116. The slowest flying speed can be read directly off the machine performance curve if this has been found by the Third Method or the Fourth Method: if the curve has been plotted, the speed can be read off the plotting: if not it can be read off the V column for the case of λ = 1·0, see page 111 or page 114. If the minimum flying speed at an altitude is required, reference must be made to the plottings of P' and P_T' on V for that altitude, as the P_T' curve sometimes cuts off the slowest part of the P' curve at great heights.

If the Third Method or Fourth Method has not been used, the only thing is to work along the λ = 1·0 line of the table of the Third Method till the V column is reached.

Throttling Curves.—A set of throttling curves scribed on celluloid is constantly wanted in performance calculations. If these are not available they should therefore be made.

Take a sheet of celluloid a little bigger than 12 inches by 8 inches. It should be about $\frac{1}{2}$ mm. to 1 mm. thick, since a moderate degree of flexibility is an advantage. Celluloid generally has a pronounced camber: it is very rarely quite flat. Lay it down then *hollow side downwards* and scribe a line 12 inches long near the bottom: this is the speed axis; also a line 8 inches long, at right angles to the first, near the *right-hand* side: this is the horse-power axis.

Now lay the sheet (still hollow side downwards) on top of the curves of page 132, axis lying on axis, and having weighted it to hold it still, trace the curves (the squares are not wanted, of course) on to the celluloid with french curves and a scriber. A compass point (a coarse one, not a needle point), or better the point belonging to a set of trammels, will make a good scriber: the point should be quite sharp but not at a fine taper, and round, not like a knife blade. No difficulty will be found in making clear scratches on the celluloid with a *light* pressure.

The curves of page 132 will cover the right-hand bottom quarter of the celluloid.

Then trace the curves of page 133 continuously with those already traced: they are the left-hand bottom quarter.

Then trace page 134 (top right-hand corner) and page 135 (top left-hand corner).

The celluloid throttling curves are now ready for use. By the way, do not attempt to rub down the burr left by the scriber with any abrasive, as that would make the celluloid difficult to see through.

In use, of course, the celluloid sheet is turned hollow side *up* so that the scribed lines are right down on the paper, the curvature of the celluloid makes it easier to press it flat on the paper, the origin is at the bottom left-hand corner, the axis of speed is along the bottom and the axis of horse-power is along the left-hand side—as all our performance curves are plotted.

Most designers plot performance curves, whether on inch or millimetre squared paper, on a moderate area, and so the 12 inch by 8 inch size of throttling curves will be all right (they can be used as they stand for performance curves plotted in any units, or any scales, and on paper divided on the English, Metric, or any system, by the way).

If a designer is in the habit of plotting performance curves on a larger area of paper than 12 inches by 8 inches, however, and intends to continue his practice, he should first plot a family

of curves from the data of the tables of pages 130 and 131 (plotting y upwards and x to the *left*) on any scale he likes, and then trace them on to celluloid as described above. Such a large sheet can be used on small-plotted performance curves too, of course.

The points in the tables which are printed in heavy type fall outside the general area of plotting, but will be useful in fixing the last part of the curves to which they refer.

Consumption and Revolutions when Throttled.—At any altitude desired take the plotting of P', P_T', and P_R' on V' and lay the celluloid throttling curves over it with speed axis on speed axis and horse-power axis on horse-power axis.

Between the minimum speed and the top speed a certain number of the throttling curves will cut the P' curve: note the values of the speeds where the intersections take place and call them V_1', V_2', V_3', etc. Note also V_{T1}', V_{T2}', etc., and V_{R1}', V_{R2}', etc., the speeds at the points where the same throttling curves cut the P_T' curve and the P_R' curve.

Then construct a table in the following form, allowing for as many horizontal lines of figures as there are intersections on the P' curve:—

V'.	V_T'.	V_R'.	$(\sigma-p)\left(\frac{V'}{V_T'}\right)^2$.	$p+(\sigma-p)\left(\frac{V'}{V_T'}\right)^2$.	$\alpha=\left[p+(\sigma-p)\left(\frac{V'}{V_T'}\right)^2\right]\frac{V'}{V_R'}$.	$\beta=\frac{V'}{V_R'}$.

Next fill up the first three columns with the values V_1', V_2', etc., V_{T1}', V_{T2}', etc., and V_{R1}', V_{R2}', etc., just determined.

Then work out the other columns.

σ can be found for the altitude in question from the curve on page 104.

p = ·161 for stationary and ·262 for rotary engines.

Then a is the ratio of the consumption per hour to the full consumption in standard density air and β is the ratio of the revolutions to the standard full revolutions.

The same method can, of course, be used for standard density air.

Best Cruising Speed at any Given Altitude.—*In a calm.*—Take the values of a and V′ from the table of page 128 and work out a column of $\frac{a}{V'}$. Plot this column against V′ and note the value of V′ which gives a minimum.

This is the best cruising speed in a calm at the altitude in question, *i.e.* the speed at which the machine will fly most miles to the gallon.

Against a Head Wind of v' Miles per Hour.—Somewhat as before, plot $\frac{a}{V' - v'}$ and note the value of V′ at which it is a minimum.

This is the best cruising speed at the altitude against the wind v'.

Doubtless it will be desired to repeat this operation for a range of wind speeds (some of them negative, *i.e.* following winds), and for a range of altitudes.

Best Cruising Altitude.—Find $\frac{a}{V'}$ as above, plot it against V′ and note the minimum value of $\frac{a}{V'}$.

Repeat the work for a series of altitudes and plot the minimum values of $\frac{a}{V'}$ against altitude. Then note the altitude at which this curve has a minimum.

This is the best cruising altitude in a calm.

The best cruising altitude against a head wind v' is got in the same way, but using $\frac{a}{V' - v'}$ instead of $\frac{a}{V'}$.

Cruising Range.—*In a Calm.*—First determine the altitude. This is probably governed by considerations bearing on navigation, the avoidance of inconvenience to passengers at high altitudes, or some such practical considerations. If not, take the best cruising altitude found above. This determines σ from the curve of page 104. Then find the best cruising speed at this altitude as above, and call it V_0'.

Now refer to the plottings of P_T', P_R', and P′ on V′ for this altitude, and, with the throttling curves, find the values of V_T' and V_R' corresponding to $V' = V_0'$ (this may involve interpolating by eye between two of the throttling curves, but that is easy enough to do).

y.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.	17.	18.
1	·13	·24	·36	·48	·60	·70	·81	·91	1·01	1·11	1·22	1·32	1·41	1·50	1·59	1·67	1·76	1·84
2	·16	·31	·46	·61	·75	·89	1·02	1·15	1·27	1·40	1·53	1·66	1·78	1·89	2·00	2·11	2·21	2·32
3	·18	·35	·52	·70	·86	1·01	1·16	1·31	1·46	1·61	1·75	1·90	2·03	2·16	2·29	2·42	2·54	2·65
4	·20	·39	·58	·77	·95	1·12	1·28	1·44	1·61	1·77	1·93	2·09	2·24	2·38	2·52	2·66	2·79	2·92
5	·21	·42	·62	·82	1·02	1·20	1·38	1·56	1·73	1·91	2·08	2·25	2·41	2·56	2·72	2·86	3·01	3·15
6	·23	·45	·66	·88	1·08	1·28	1·47	1·65	1·84	2·02	2·21	2·39	2·56	2·73	2·89	3·04	3·19	3·34
7	·24	·47	·70	·92	1·14	1·34	1·54	1·74	1·94	2·13	2·33	2·52	2·70	2·87	3·04	3·20	3·36	3·52
8	·25	·49	·73	·96	1·19	1·41	1·61	1·82	2·02	2·23	2·43	2·63	2·82	3·00	3·18	3·35	3·52	3·68

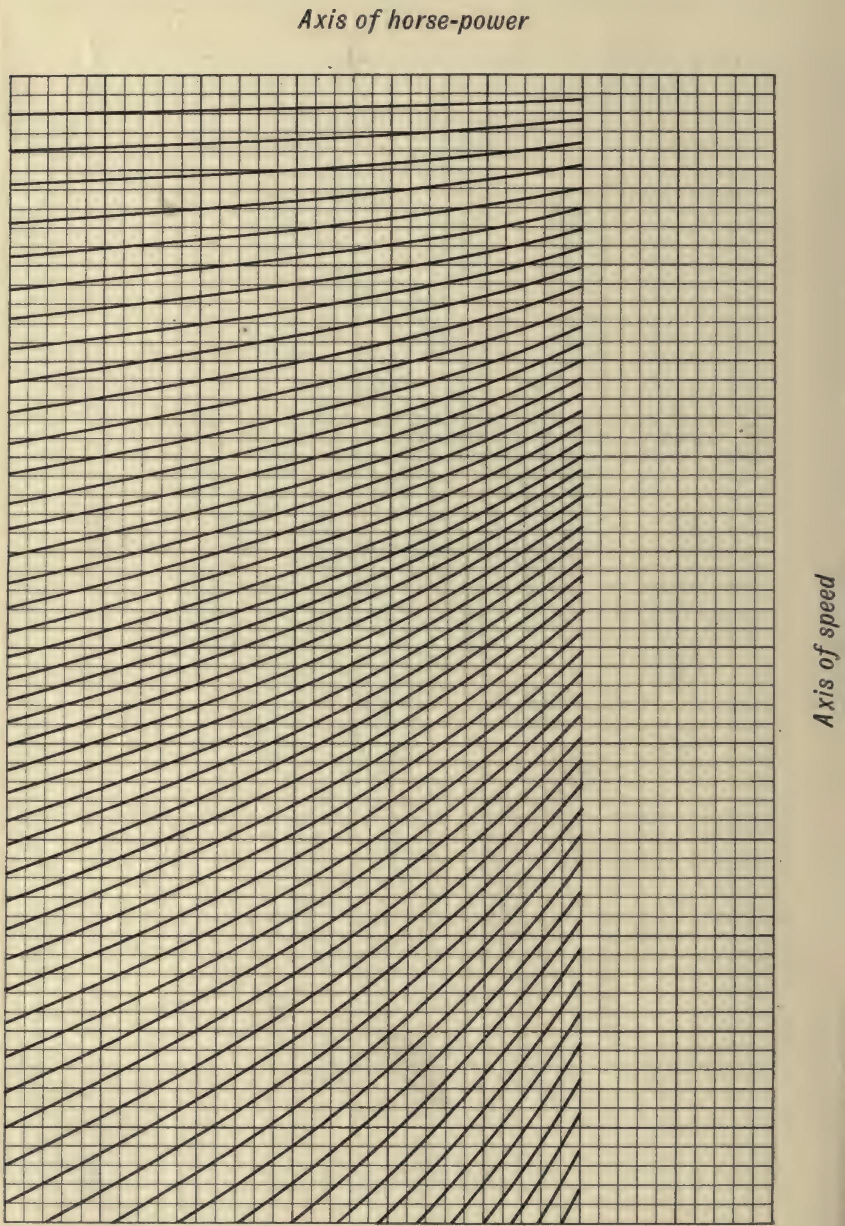
y.	19.	20.	21.	22.	23.	24.	25.	26.	27.	28.	29.	30.	31.	32.	33.	34.	35.	36.
1	1·92	2·00	2·07	2·14	2·21	2·29	2·37	2·45	2·54	2·63	2·72	2·82	2·92	3·02	3·13	3·24	3·35	3·47
2	2·42	2·51	2·60	2·69	2·79	2·89	2·99	3·09	3·20	3·31	3·43	3·55	3·68	3·80	3·94	4·08	4·22	4·37
3	2·77	2·88	2·98	3·08	3·19	3·30	3·42	3·54	3·66	3·79	3·93	4·06	4·21	4·36	4·51	4·67	4·83	5·00
4	3·05	3·17	3·28	3·39	3·51	3·64	3·76	3·90	4·03	4·18	4·32	4·47	4·63	4·79	4·96	5·14	5·32	5·50
5	3·28	3·41	3·53	3·66	3·78	3·92	4·05	4·20	4·35	4·50	4·65	4·82	4·99	5·16	5·35	5·53	5·73	5·93
6	3·49	3·63	3·75	3·88	4·02	4·16	4·31	4·46	4·62	4·78	4·95	5·12	5·30	5·49	5·68	5·88	6·09	6·30
7	3·67	3·82	3·95	4·09	4·23	4·38	4·54	4·70	4·86	5·03	5·21	5·39	5·58	5·78	5·98	6·19	6·41	6·63
8	3·84	3·99	4·13	4·28	4·43	4·58	4·74	4·91	5·08	5·26	5·45	5·64	5·83	6·04	6·25	6·47	6·70	6·93

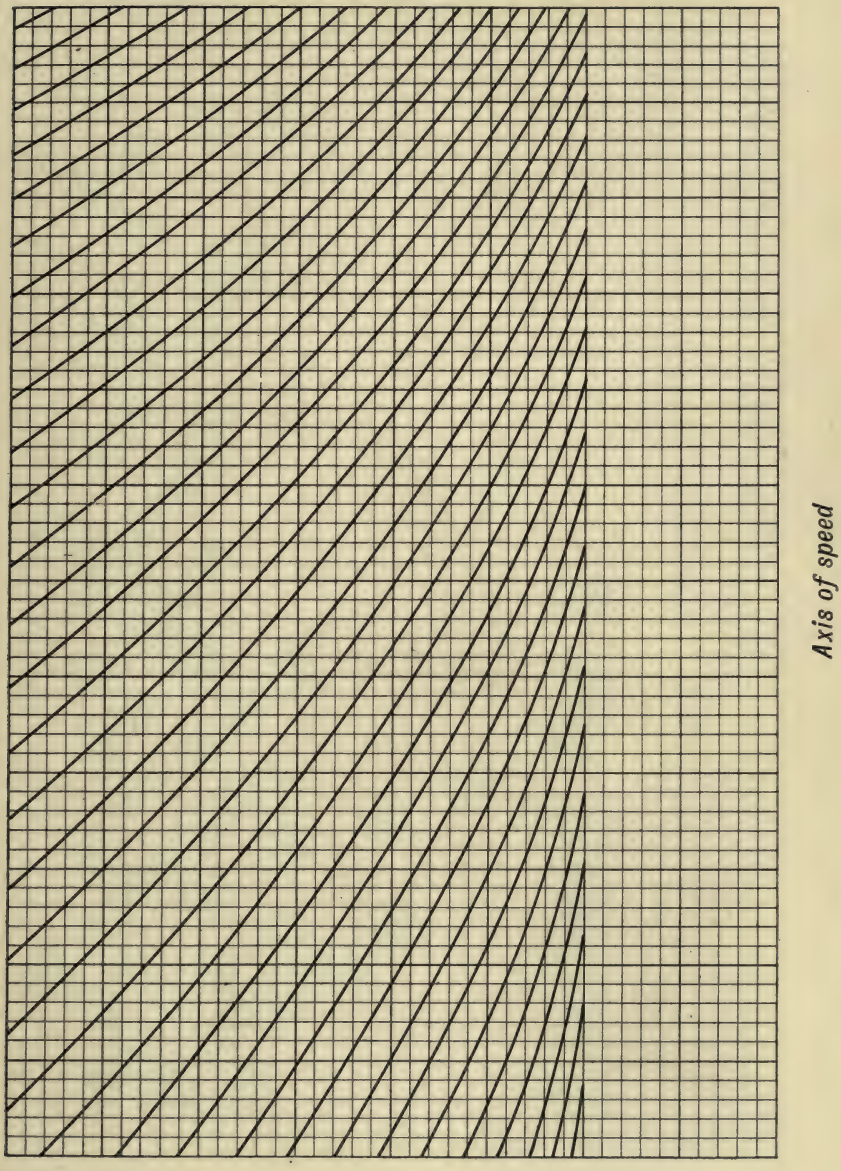
Values of x.

y.	37.	38.	39.	40.	41.	42.	43.	44.	45.	46.	47.	48.	49.	50.	51.	52.	53.	54.
1	3·59	3·72	3·85	3·98	4·12	4·27	4·42	4·57	4·73	4·90	5·07	5·25	5·43	5·62	5·82	6·03	6·24	6·46
2	4·52	4·68	4·85	5·02	5·19	5·37	5·56	5·76	5·96	6·17	6·39	6·61	6·84	7·09	7·33	7·59	7·86	8·13
3	5·18	5·36	5·55	5·74	5·94	6·15	6·37	6·59	6·82	7·06	7·31	7·57	7·84	8·11	8·40	8·69	9·00	9·31
4	5·70	5·90	6·11	6·32	6·54	6·77	7·01	7·26	7·51	7·77	8·05	8·33	8·62	8·93	9·24	9·57	9·90	10·25
5	6·14	6·35	6·58	6·81	7 05	7·29	7·55	7·82	8·09	8·38	8·67	8·97	9·29	9·62	9·95	10·30	10·67	11·04
6	6·52	6·75	6·99	7·23	7·49	7·75	8·02	8·31	8·60	8·90	9·21	9·54	9·87	10·22	10·58	10·95	11·33	11·73
7	6·87	7·11	7·36	7·62	7·88	8·16	8·45	8·74	9·05	9·37	9·70	10·04	10·39	10·76	11·14	11·53	11·93	
8	7·18	7·43	7·69	7·96	8·24	8·53	8·83	9·14	9·46	9·80	10·14	10·50	10 87	11·25	11·64	**12·05**		

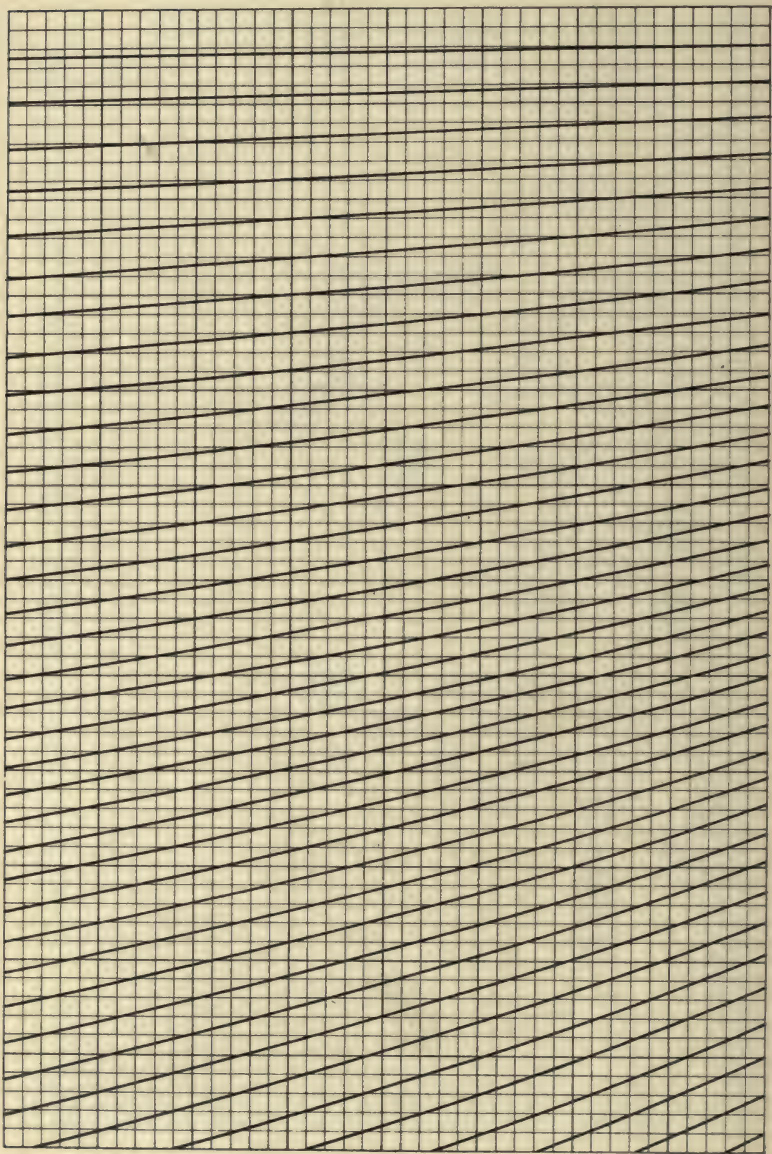
y.	55.	56.	57.	58.	59.	60.	61.	62.	63.	64.	65.	66.	67.	68.	69.	70.	71.	
1	6·68	6·92	7·16	7·41	7·67	7·94	8·22	8·51	8·81	9·12	9·44	9·77	10·12	10·47	10·84	11·22	11·62	
2	8·42	8·72	9·02	9·34	9·67	10·03	10·36	10·72	11·10	11·49	11·89	**12·31**	**12·75**	**13·19**	**13·66**	**14·14**	**14·63**	
3	9·64	9·98	10·33	10·69	11·07	11·46	11·86	**12·28**	**12·71**									
4	10·61	10·98	11·37	11·77	**12·18**													
5	11·43	11·83																
6																		
7																		
8																		

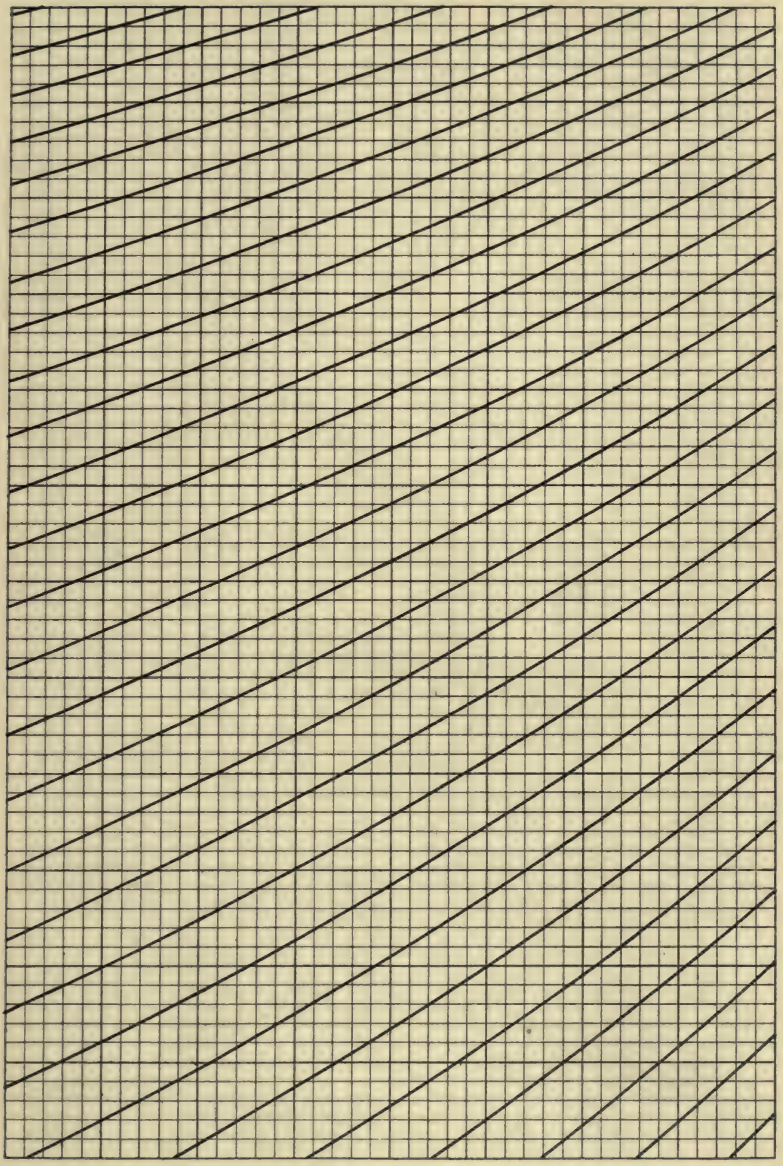
Values of x.


Axis of horse-power
Axis of speed


Axis of speed

Axis of horse-power





Let W_0 be the total weight of the machine at the commencement of the flight (that is the value of W used in finding the machine performance curve) and let W be the weight of the machine when she has flown till her tanks are dry.

$p = \cdot 161$ for stationary, and $\cdot 262$ for rotary engines.

Δ is the full consumption of the engine in pounds per hour (*not* pounds per B.H.P. hour) in standard density air.

$$a = \frac{\Delta(\sigma - p)V_0'^2}{W_0 V_R' V_T'^2}$$

$$w = W + \frac{pV_T'^2 W_0}{(\sigma - p)V_0'^2}$$

$$w_0 = W_0\left[1 + \frac{pV_T'^2}{(\sigma - p)V_0'^2}\right].$$

Then the cruising range in miles is

$$\frac{2\cdot 303}{a} \log_{10}\left(\frac{w_0}{w}\right)$$

but to secure the full mileage, if W' is the weight of the machine at any time during the trip, the machine must be flown at the speed in miles per hour, of

$$V_0'\sqrt{\frac{W'}{W_0}}.$$

Against a Head Wind of v' Miles Per Hour.—First determine the altitude, either from practical considerations or, if these do not prevent it, by the preceding method for finding the best cruising altitude against the given wind.

Now find V_0', V_T', and V_R' for this altitude by a similar process to that used for the calm.

Then a, w, and w_0 have the same values as above.

Let x be the air miles to dry tanks, and t the time in hours to dry tanks, then

$$x = \frac{2\cdot 303}{a} \log_{10}\left(\frac{w_0}{w}\right)$$

$$b = \sqrt{\frac{pV_T'^2 W_0}{(\sigma - p)V_0'^2}}$$

$$t = \frac{\cdot 0349\sqrt{W_0}}{baV_0'} \tan^{-1}\left\{\frac{b(\sqrt{W_0} - \sqrt{W})}{b^2 + \sqrt{W_0 W}}\right\}$$

the angle being in degrees. An ordinary table of tangents can be used to hunt up the angle in.

Then the cruising distance in miles is

$$x - v't,$$

but again, to secure the full mileage, the machine must be flown at a series of air speeds—

$$V_0'\sqrt{\frac{W'}{W_0}}$$

as W' gradually decreases from W_0 to W.

If the cruising revolutions are required, see Chapter V., page 59.

Examples.—For numerical examples illustrating the methods of this Chapter, see Chapter XIX., page 164, and Chapter XXII., page 189, etc.

Theory.—For theoretical explanations see Chapter V., page 35.

CHAPTER XIII.

GROUND PERFORMANCE.

Getting off a Deck.—Turn to the curve of P_T on V and decide on a speed about three-quarters of the estimated minimum flying speed : a reasonable guess will be near enough. Call this speed V miles per hour. Note the value of P_T in horse-power corresponding to this speed. Call it P_T. Then T is defined as

$$T = 12{,}070 \frac{P_T}{V}.$$

Now decide on the value of λ for the run along the deck : this can be got from the model tests and the angle of incidence desired. This angle of incidence may be determined by practical considerations : if not, take the angle which gives minimum wing resistance.

λ, $k_{L max}$, S, R_1, R_2, W, d, S′, L/D are all as defined on page 110.

$$a = 11{\cdot}4\lambda k_{L max} T$$

$$K = \frac{{\cdot}001495 a S R_1}{W d^2 - a S'} + {\cdot}001495(R_1 + R_2) + \frac{{\cdot}07625 \lambda k_{L max} S W d^2}{L/D(W d^2 - a S')}.$$

$$B = \frac{1256 T}{d^2 V^2}$$

$$b = \sqrt{B + 1} - 1.$$

Now turn to the side view drawing of the machine and set it at the stalling angle (taken from model tests) so that the relative wind speed is now horizontal.

From the centre of the propeller boss lay in a line aft and horizontal, whose length represents, on any convenient scale, the estimated minimum flying speed which is $\frac{4}{3}$V.

Also measure off $\frac{4}{3}b$V aft along the propeller axis.

Combine these two velocities by the parallelogram of velocities, and note the point A where the resultant meets the line joining the top and bottom leading edges of the machine. Also note the angle of this resultant to the wing chord.

Now turn to the model tests and note the lift coefficient k_L' corresponding to this angle.

Then turn to the front-view drawing of the machine and mark the point A. Then with centre A and radius $\frac{1}{2}d$ describe a circle.

Now find S″, using this circle, in the same way as on page 110 S′ was found, using the propeller circle.

$$v_m = \sqrt{\frac{422\mathrm{W}}{k_{\mathrm{L}max}(\mathrm{S} - \mathrm{S}'') + k_{\mathrm{L}}'(1 + b)^2\mathrm{S}''}}$$

$$\beta = \sqrt{\frac{T}{\mathrm{K}}}.$$

Then the length run on the deck in feet to get off is

$$\frac{1\cdot151\mathrm{W}}{\mathrm{K}}\left[\left(1 + \frac{v_w}{\beta}\right)\log_{10}\left(\frac{\beta + v_w}{\beta + v_m}\right) + \left(1 - \frac{v_w}{\beta}\right)\log_{10}\left(\frac{\beta - v_w}{\beta - v_m}\right)\right]$$

where $$v_w = 1\cdot467\mathrm{V}_w$$

and V_w is the speed of the head wind in miles per hour relative to the deck.

Getting Off the Ground.—Using the same notation as above, excepting that V_w does not occur, the length of run to get off the ground in still air in feet is

$$\frac{1\cdot151\mathrm{W}}{\mathrm{K}}\left[\log_{10}\left(\frac{\beta}{\beta + v_m}\right) + \log_{10}\left(\frac{\beta}{\beta - v_m}\right)\right].$$

Landing on a Deck.—

$$\mathrm{K} = \cdot001495(\mathrm{R}_1 + \mathrm{R}_2) + \frac{\cdot07625k_{\mathrm{L}max}\mathrm{S}}{\mathrm{L/D}},$$

R_1, R_2, $k_{\mathrm{L}max}$, and S having their usual significances, while L/D is the value corresponding to $\lambda = 1\cdot0$.

V_m is the slowest flying speed in miles per hour found on page 126.

Then the length run along the deck in feet on landing is

$$\frac{\mathrm{W}}{\mathrm{K}}\left[2\cdot303\log_{10}\left(\frac{\mathrm{V}_m}{\mathrm{V}_w}\right) - \frac{\mathrm{V}_m - \mathrm{V}_w}{\mathrm{V}_m}\right]$$

where V_w is the speed of the head wind in miles per hour relative to the deck.

Landing on the Ground.—Let the effective coefficient of friction between the machine and the ground be μ, *i.e.* μ is the ratio of the ground resistance to the part of the weight of the machine borne by the ground when the machine is on her wheels and tail skid.

V_m is the alighting speed in miles per hour; it is either the slowest flying speed found on page 126, or the landing speed

on glide found on page 116, according to the type of landing used.

K has the same value as for landing on a deck—

$$a = \frac{K}{W} - \frac{14 \cdot 95 \mu}{V_m^2}.$$

Then the length run in feet to pull up on landing in still air is—

$$\frac{1 \cdot 151}{a} \log_{10} \left(\frac{KV_m^2}{14 \cdot 95 W \mu} \right)$$

but if a happens to be zero we must use instead for the length of landing run in feet—

$$\frac{V_m^2}{29 \cdot 9 \mu}.$$

Examples.—For numerical examples illustrating the methods of this chapter, see Chapter XX., page 176.

Theory.—For theoretical explanations see Chapter VI., page 61.

CHAPTER XIV.

WATER PERFORMANCE.

The Condition for Getting Off the Water.—*When a Model Test has been Made Particularly for the Machine.*—If the test results are quoted in knots, first convert them to miles per hour by the formula

$$V = 1{\cdot}151\ \mathbf{V}$$

where $\mathbf{V}$ is a speed in knots and V is the same speed in miles per hour.

If the results are given in the form of a table of values of F, the water resistance of the machine in pounds for a series of values of V, the speed in miles per hour, well and good. If, however, the results are given in the form of a table of values f, the water resistance of the model in pounds for a series of values of v, the speed in miles per hour, we must first convert to F and V by the formulæ—

$$F = \left(\frac{L}{l}\right)^3 f$$

and

$$V = \sqrt{\frac{L}{l}}\,v,$$

where $\frac{L}{l}$ is the linear ratio of dimensions of the machine to the model.

Now the minimum flying speed has been used in making the model tests: let it be V_1 miles per hour and let the corresponding horse-power on the machine performance curve be P_1.

Now draw up a table in the following form, allowing for as many horizontal lines of figures as test speeds are available :—

V.	F.	$P_1\left(\frac{V}{V_1}\right)^3$.	$\frac{VF}{375}$.	$P_w = P_1\left(\frac{V}{V_1}\right)^3 + \frac{VF}{375}$.

Then fill up the first two columns with the data in hand, work out the other columns, and plot P_w, the total water horsepower on V. Plot P_T on V on the same paper.

If the curves do not cut, the machine will get off in a calm. If they do cut, she will not. The condition for getting off in a wind is easier than that for getting off in a calm, so it need not be considered.

The point V_1, P_1 is the last point on the P_w curve, of course.

When a Special Model Test is not Available.—In this case the float system should have been designed geometrically similar to a known system, tests of which are available, and should be fixed on the machine at the same angle.

With the same notation as before, let capital letters refer to the new design, italicised capitals to the full scale machine being copied from, and small letters to the model tests of the latter.

Then the similarity is only correct if, in addition to geometrically similar lines and identity of angular setting we have—

$$\frac{\mathrm{V}_1}{V_1} = \sqrt{\frac{\mathrm{L}}{L}}$$

and

$$\frac{\mathrm{W}}{W} = \left(\frac{\mathrm{L}}{L}\right)^3.$$

If *all* these conditions are not fulfilled, the designer is working in the dark, and can only rely on his judgment to determine if the machine will come off in a calm. There are cases on record where designers have so relied on their judgment and have been wrong.

If, however, *all* the above conditions are satisfied, and we have a table of f and v, we can convert it to a table of F and V by the formulæ—

$$\mathrm{F} = \left(\frac{\mathrm{L}}{l}\right)^3 f$$

and

$$\mathrm{V} = \sqrt{\frac{\mathrm{L}}{l}}\,v$$

while if, instead, we have a table of F and V, we can convert it to a table of F and V by the formulæ—

$$\mathrm{F} = \left(\frac{\mathrm{L}}{L}\right)^3 F$$

and

$$\mathrm{V} = \sqrt{\frac{\mathrm{L}}{L}}\,V.$$

Then in either case we can apply the same method as before to find the P_w curve and the getting off criterion.

Examples.—For numerical examples illustrating the methods of this chapter, see Chapter XXI., page 179.

Theory.—For theoretical explanations see Chapter VII., page 71.

PART III.

ILLUSTRATIVE EXAMPLES.

CHAPTER XV.

BODY RESISTANCE.

Example (1).—*Resistance of a Fuselage.*—Take the case of a fuselage whose maximum cross section is a rectangle, 26 inches wide by 36 inches deep, with an arched top rising 5 inches at the centre.

First find the area of cross section by using a planimeter on the drawing of the cross section. This gives $a = 7{\cdot}07$ square feet.

$\therefore\ r = 2{\cdot}5 \times 7{\cdot}07 = 17{\cdot}7$ pounds at 100 miles per hour.

The resistance of windscreen, cockpit, etc., is not included in the above.

Example (2).—*Resistance of a Nacelle.*—Take a nacelle of the same cross section as above. Then (again, of course, excluding windscreen, etc.)

$r = 7 \times 7{\cdot}07 = 49{\cdot}5$ pounds at 100 miles per hour.

Example (3).—*Resistance of an Engine Egg.*—Take a circular egg of 43 inches diameter (such as would be used for a large stationary radial engine). Then, excluding the resistance of projecting cylinder heads or engine cooling louvres, since the area is 10·09 square feet,

$r = 7 \times 10{\cdot}09 = 70{\cdot}6$ pounds at 100 miles per hour.

Example (4).—*Resistance of a Flying Boat Hull, Including the Steps.*—Consider the case of a flying boat hull whose maximum cross section is an egg-shaped oval 3 feet 6 inches beam by 3 feet 11 inches deep; further we will suppose that the egg shape is not a very great departure from elliptical form, then we can save ourselves the trouble of using the planimeter by taking the elliptic formula

$$a = 3{\cdot}5 \times 3{\cdot}918 \times \frac{\pi}{4} = 10{\cdot}75 \text{ square feet.}$$

Therefore the resistance of the hull without cockpits or steps is

$1{\cdot}3 \times 10{\cdot}75 = 14{\cdot}0$ pounds at 100 miles per hour.

Now suppose that there are two steps, of which the front one has a maximum depth of 2 inches, and a beam of 4 feet, and the

back one a depth of 5 inches, and a beam of 1 foot 6 inches. We find, however, we suppose, that the steps project to form a wide "mudguard," so that the sum of the areas of the steps when found by the planimeter comes out at 1·79 square feet.

We now turn to page 83 and refer to the formula $r = 29a$ there given. Therefore the resistance of the steps is $29 \times 1·79 = 51·9$ pounds at 100 miles per hour.

Therefore the resistance of the hull, including steps but without cockpit, etc., is given by

$$r = 14·0 + 51·9 = 65·9 \text{ pounds at 100 miles per hour.}$$

Example (5).—*Resistance of a Pontoon Float.*—Consider a float 3 feet 6 inches beam by 2 feet 6 inches deep at the maximum cross section. Then the maximum cross sectional area is 8·75 square feet.

$$\therefore r = 6 \times 8·75 = 52·5 \text{ pounds at 100 miles per hour.}$$

Example (6).—*Resistance of a Complete Tail Unit.*—Suppose the area of the tail plane plus elevators is 66 square feet and the area of the fin and rudder together is 21 square feet. Then the resistance of the complete tail unit is given by

$$r = ·78 \times 66 + ·58 \times 21 = 51·5 + 12·2$$
$$= 63·7 \text{ pounds at 100 miles per hour.}$$

Example (7).—*Resistance of Struts.*—Take the case of 20 feet of strut $1\frac{1}{2}$ inches wide across the wind direction, 20 feet $2\frac{1}{2}$ inches wide and 14 feet 1 inch wide.

$$\therefore a = 30 + 50 + 14 = 94 \text{ inch-feet units.}$$

We will suppose that all the struts have cross sections geometrically similar to Fig. 1, page 149.

First round off the tail a bit and we get Fig. 2. Note the fineness ratio of Fig. 2, which is 3·69.

Then expand Fig. 2 to 4 inches by 1 inch and compare with shapes A and B of page 80. For convenience these shapes are repeated in Figs. 3 and 4 respectively, the position taken up by the traced contour being shown dotted.

On considering Figs. 3 and 4, remembering that after all it is only tails that count, we conclude that the strut we are dealing with is $\frac{1}{3}$ the way from shape B towards shape A. With this fact and the fineness ratio of 3·69, we get from the curve of page 80

$$x = ·268.$$

$$\therefore r = 94 \times ·268 = 25·2 \text{ pounds at 100 miles per hour.}$$

Example (8).—*Interference of Struts.*—Suppose that there are on each side of the machine a pair of struts 5 feet long, of

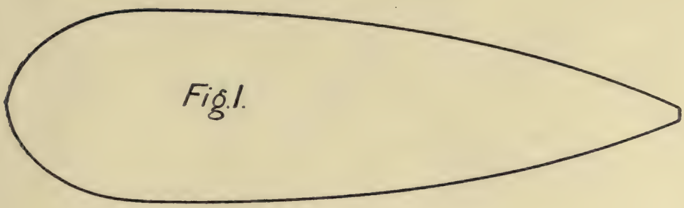


Fig.1.

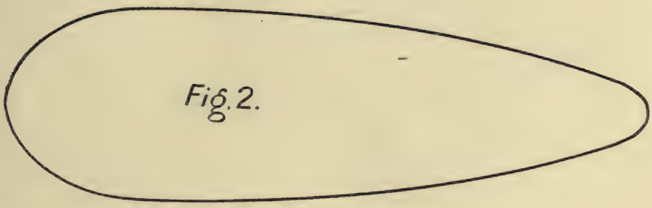


Fig.2.

Fig.3.

Fig.4.

the same type of cross section as in example (7), side by side, with their centres $4\frac{1}{2}$ inches apart, the struts of each pair being, one 1 inch thick, and one 2 inches thick.

Then as the shape is the same as in example (7), x is the same.

$$\therefore x = \cdot 268.$$

Also the total area is $a = 2 \times 5(2 + 1) = 30$ inch-feet units.

Now the centres distance divided by the mean width is

$$\frac{4\frac{1}{2}}{1\frac{1}{2}} = 3.$$

Therefore the correction is $1 \cdot 194$. Therefore the actual total resistance of the four struts is given by

$r = 30 \times \cdot 268 \times 1 \cdot 194 = 9 \cdot 6$ pounds at 100 miles per hour.

Example (9).—*Resistance of Stream-line Wires.*—Find the resistance of five stream-line wires of thread sizes $\frac{1}{4}$ inch, $\frac{1}{4}$ inch, $\frac{3}{8}$ inch, $\frac{3}{8}$ inch, and $\frac{3}{8}$ inch respectively, and lengths 100, 95, 75, 70, and 68 inches respectively.

l.	d'.	$250d'$.	$l+250d'$.	$\cdot 025d'(l+250d')$.
100	$\frac{1}{4}$	62·5	162·5	1·01
95	$\frac{1}{4}$	62·5	157·5	·99
75	$\frac{3}{8}$	93·7	168·7	1·58
70	$\frac{3}{8}$	93·7	163·7	1·53
68	$\frac{3}{8}$	93·7	161·7	1·52
			Total r =	6·6

Therefore the resistance of the batch is given by

$$r = 6 \cdot 6 \text{ pounds at 100 miles per hour.}$$

Example (10).—*Resistance of Cable and Piano Wire.*—Consider a batch consisting of a 10 cwt. cable 150 inches long, a 20 cwt. cable 100 inches long, and a 10 gauge piano wire 50 inches long. After looking up the diameters we find the resistances are—

$$\cdot 26 \times \cdot 115(150 + 200 \times \cdot 115) = 5 \cdot 18$$
$$\cdot 26 \times \cdot 150(100 + 200 \times \cdot 150) = 5 \cdot 07$$
$$\cdot 21 \times \cdot 128(50 + 300 \times \cdot 128) = 2 \cdot 38$$

Total $r = 12 \cdot 6$ pounds at 100 miles per hour.

Example (11).—*Resistance of Wheels.*—Consider two 800 by 150 wheels. Without fairing—

$r = 2 \times \cdot 000184 \times 800 \times 150 = 44 \cdot 2$ pounds at 100 miles per hour.

With shields to the rims only—

$r = 2 \times \cdot 000113 \times 800 \times 150 = 27 \cdot 1$ pounds at 100 miles per hour.

While with the complete shields from tyre to tyre—

$r = 2 \times \cdot 000062 \times 800 \times 150 = 14 \cdot 9$ pounds at 100 miles per hour.

Example (12).—*Resistance of a Cockpit.*—Suppose the cockpit is 2 feet wide and 3 feet long at the opening, then the resistance is given by

$$r = \cdot 7 \times 24 = 16 \cdot 8 \text{ pounds at 100 miles per hour.}$$

Example (13).—*Resistance of a Windscreen.*—Take a windscreen 8 inches by 12 inches, then the resistance is given by

$$r = 29 \times \tfrac{2}{3} = 19 \cdot 3 \text{ pounds at 100 miles per hour.}$$

Example (14).—*Resistance of Engine Cylinders.*—Take the case of a 9-cylinder stationary radial engine with the cylinders, which are six inches diameter over the gills, projecting 4 inches beyond the cowling, then the total area of cylinder can be taken as $a = 9 \times \frac{1}{2} \times \frac{1}{3} = 1 \cdot 5$ square feet.

$$\therefore\ r = 31 \times 1 \cdot 5 = 46 \cdot 5 \text{ pounds at 100 miles per hour.}$$

Example (15).—*Line of Action of Body Resistance.*—Let us assume the following particulars: Take the top fuselage longeron as datum; reckon a's in inches and positive for parts above the longeron; use the batches defined on page 83; let the values of a's and R's be as in the table:—

Batch.	R_1, etc.	a_1, etc.	a_1R_1, etc.
(1)	30	+ 6	+ 180
(2)	13	− 48	− 624
(3)	12	− 36	− 432
(4)	107	− 10	− 1070
(5)	21	0	0
(6)	7	+ 14	+ 98
Total R =	190	Total	− 1848

Therefore the line of action of the body resistance is at a height above the top longeron of

$$\frac{-\ 1848}{190} = -\ 9 \cdot 7 \text{ inches,}$$

i.e. it is 9·7 inches below the top longeron.

CHAPTER XVI.

WING CHARACTERISTICS.

Example.—We will take an overall span of 36 feet, of which 2 feet 6 inches is rounded off in plan at each tip: gap, 5 feet; chord, 6 feet; stagger, 3°; R.A.F. 15 section. We will take the model test data for this wing given on page 87.

Then

$$\text{aspect ratio} = \tfrac{36}{6} = 6;$$
$$\text{gap/chord} = \tfrac{5}{6} = \cdot 834;$$
$$\text{stagger} = 3^\circ;$$
$$\text{wing tip rounding} = \frac{2\cdot 5}{6} = \cdot 417 \text{ of chord};$$
$$\text{dimensions} = 120.$$

We can now fill up, with the aid of pages 88 to 97 the following table:—

λ.	Corrections for					Model L/D.	Corrected L/D.
	Aspect Ratio.	Gap/Chord.	Stagger.	Wing Tips.	Dimensions.		
·1	1·000	·980	1·009	1·000	1·084	7·00	7·50
·2	1·000	·930	1·009	1·027	1·026	12·59	12·45
·3	1·000	·895	1·006	1·039	1·048	16·25	15·92
·4	1·000	·843	1·001	1·052	1·080	17·25	16·54
·5	1·000	·791	1·001	1·068	1·065	16·75	15·06
·6	1·000	·780	·997	1·048	1·016	15·83	13·10
·7	1·000	·771	1·001	1·081	1·031	14·59	12·55
·8	1·000	·753	1·001	1·081	1·009	13·11	10·79
·9	1·000	·742	1·001	1·027	1·006	11·65	8·94
1·0	1·000	·742	1·001	1·115	1·000	7·47	6·19
k_{Lmax}	1·000	·908	1·006	·993	1·025	·513	·4770

The figures in the last column are now in the form required for use in machine performance calculations.

CHAPTER XVII.

PROPELLER PERFORMANCE CURVES.

Example (1).—*Diameter of a Two-bladed Propeller.*—

Suppose
$$H = 360,$$
$$n_0 = 1100,$$
$$V_0 = 80,$$
$$\therefore\ d = 10{,}000\sqrt[4]{\frac{360}{53{\cdot}5 \times 1100^2 \times 80}} = 162 \text{ inches.}$$

Example (2).—*Diameter of a Four-bladed Propeller.*—

Suppose
$$H = 360,$$
$$n_0 = 1100,$$
$$V_0 = 80,$$
$$\therefore\ d = 10{,}000\sqrt[4]{\frac{360}{111 \times 1100^2 \times 80}} = 135 \text{ inches.}$$

Example (3).—*Propeller Performance Curves in Standard Density Air.*—

Suppose
$$V_0 = 80,$$
$$d = 135,$$
$$n_0 = 1100,$$
$$H = 360,$$
$$\therefore\ J = \frac{80}{1100 \times 135} = {\cdot}000538.$$

$\frac{V}{V_0}$.	η_T.	η_R.	V.	P_T.	P_R.
·4	·400	·435	32	144	156
·6	·540	·592	48	194	213
·8	·648	·694	64	233	250
1·0	·740	·740	80	266	266
1·2	·803	·723	96	289	260
1·4	·855	·650	112	308	234

The curves of P_T and P_R on V as a base will be found plotted on page 170 in connection with another example.

Example (4).—*Performance of the Same Propeller at* 10,000, 20,000, 30,000, *and* 40,000 *feet.*

The engine being a stationary one, p = ·161 and q = ·839, therefore we can draw up the following tables:—

Altitude.	σ.	$\sigma - p$.	$\sigma_1 = \frac{\sigma - p}{q}$.
10,000	·740	·579	·690
20,000	·530	·369	·440
30,000	·368	·207	·248
40,000	·242	·081	·0965

Altitude.		10,000.		20,000.		30,000.		40,000.	
V.	P_T.	V′.	P_T'.	V′.	P_T'.	V′.	PT′.	V′.	P_T'.
32	144	30·9	96	29·2	58	26·3	29	20·2	9
48	194	46·4	129	43·7	78	39·4	39	30·3	12
64	233	61·8	155	58·3	93	52·6	47	40·4	14
80	266	77·2	177	72·9	106	65·7	54	50·5	16
96	289	92·7	192	87·4	116	78·8	59	60·6	17
112	308	108·1	205	102·0	123	91·9	62	70·7	19

Altitude.		10,000.		20,000.		30,coo		40,000	
V.	P_R.	V′.	P_R'.	V′.	P_R'.	V′.	P_R'.	V′.	P_R'.
32	156	32	115	32	83	32	57	32	38
48	213	48	157	48	113	48	78	48	51
64	250	64	185	64	132	64	92	64	60
80	266	80	197	80	141	80	98	80	64
96	260	96	192	96	138	96	96	96	63
112	234	112	173	112	124	112	86	112	57

CHAPTER XVIII.

MACHINE PERFORMANCE CURVE.

Example (1).—*This Example shows the First Method of Obtaining a Machine Performance Curve.*—

Suppose $W = 5700,$

$$S = 695, \qquad \therefore\ a = \frac{196 \times 5700}{695 \times \cdot 590} = 2725,$$

$$k_{L\,max} = \cdot 590,$$

$$R = 510, \qquad \beta = \frac{510 \times 2725}{10^4} = 139.$$

λ.	L/D.	$\frac{W}{L/D}$.	$\frac{\beta}{\lambda}$.	$T = \frac{W}{L/D} + \frac{\beta}{\lambda}$.	$V = \sqrt{\frac{a}{\lambda}}$.	$P = \frac{VT}{375}$.
·1	6·42	888	1390	2278	165·0	1002
·2	11·41	499	695	1194	116·7	372
·3	15·65	364	463	827	95·4	210
·4	17·63	323	347	670	82·6	148
·5	16·15	353	278	631	73·9	124
·6	13·81	413	232	645	67·4	116
·7	13·03	437	199	636	62·4	106
·8	11·33	503	174	677	58·4	105
·9	9·64	591	154	745	55·0	109
1·0	8·05	708	139	847	52·2	118

The values of P and V are plotted on page 157, and the full line curve is drawn through them, the opportunity being taken to fair the curve slightly where the points are irregular.

Example (2).—*This Example shows the Second Method.*—

Suppose $W = 5700$, $\therefore\ a = \dfrac{196 \times 5700}{695 \times {\cdot}590} = 2725$,

$S = 695$,

$k_{L_{max}} = {\cdot}590$, $\beta = \dfrac{510 \times 2725}{10^4} = 139$,

$R = 510$,

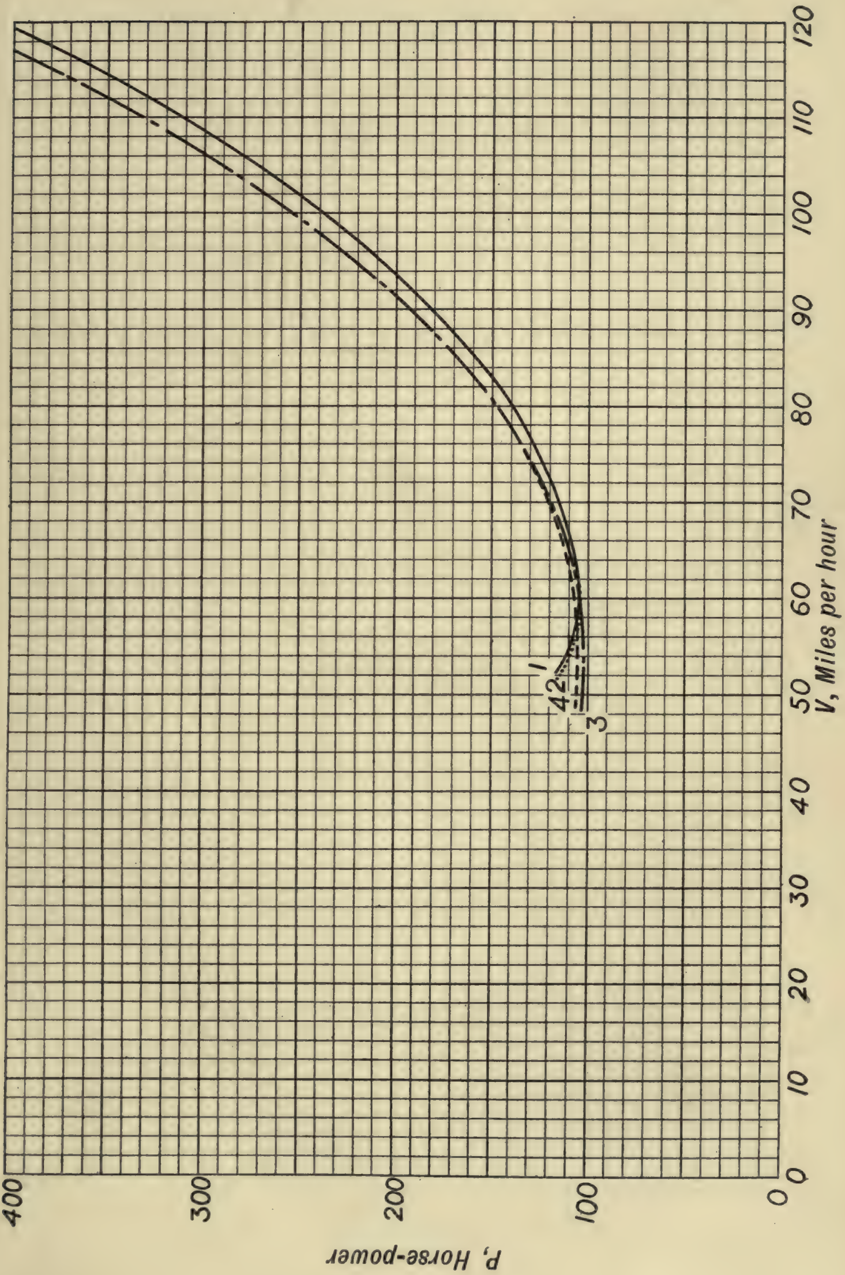
$c = 7$, $\gamma = \dfrac{14}{7} = 2$,

$l = 14$,

$l' = 16{\cdot}1$, $\delta = \dfrac{16{\cdot}1}{7} = 2{\cdot}3$.

λ.	L/D.	k_c.	$\delta - k_c$.	$L/W = \dfrac{\gamma}{\delta - k_c}$.	$\dfrac{W}{L/D}$.	$\dfrac{\beta}{\lambda}$.	$\epsilon = \dfrac{W}{L/D} + \dfrac{\beta}{\lambda}$.	$V = \sqrt{\dfrac{\alpha\, L/W}{\lambda}}$.	$P = \dfrac{\epsilon V\, L/W}{375}$.
·1	6·42	·650	1·650	1·212	888	1390	2278	181·8	1337
·2	11·41	·450	1·850	1·081	499	695	1194	121·4	418
·3	15·65	·377	1·923	1·040	364	463	827	97·2	223
·4	17·63	·341	1·959	1·021	323	347	670	83·5	152
·5	16·15	·319	1·981	1·010	353	278	631	74·3	126
·6	13·81	·302	1·998	1·001	413	232	645	67·5	116
·7	13·03	·291	2·009	·996	437	199	636	62·3	105
·8	11·33	·287	2·013	·994	503	174	677	58·2	104
·9	9·64	·280	2·020	·991	591	154	745	54·8	108
1·0	8·05	·276	2·024	·998	708	139	847	51·8	116

The values of P and V are plotted on page 157 and the curve of small dots is drawn through them, the opportunity being taken to fair the curve slightly where the points are irregular. For most of its length this curve is indistinguishable from the full line curve.


V, Miles per hour
0
10
20
30
40
50
60
70
80
90
100
110
120
P, Horse-power
400
300
200
100
0
1
2
3
4

Example (3).—*This Example Shows the Third Method.*—

Suppose $W = 5700$, $S = 695$, $S' = 95$, $k_{L max} = \cdot 590$, $R_1 = 135$, $R_2 = 375$, $c = 7$, $l = 14$, $l' = 16\cdot 1$, $d = 150$,

$$\therefore\ a' = 1 - \frac{7\cdot 2 \times 135}{150^2} = 1 - \cdot 0432$$
$$= \cdot 9568,$$
$$b' = \frac{510}{10^4} = \cdot 051,$$
$$c' = \frac{367 \times \cdot 590 \times 95}{150^2} = \cdot 914,$$
$$d' = \cdot 0051 \times \cdot 590 \times 695 = 2\cdot 09,$$
$$a = \frac{5700}{\cdot 9568 \times 2\cdot 09 + \cdot 051 \times \cdot 914}$$
$$= \frac{5700}{2\cdot 00 + \cdot 047} = \frac{5700}{2\cdot 047} = 2784,$$
$$\gamma = \frac{14}{7} = 2,$$
$$\delta = \frac{16\cdot 1}{7} = 2\cdot 3.$$

Now see table on next page.

The values of P and V are plotted on page 157, and the chain dotted curve is drawn through them, the opportunity being taken to fair the curve slightly where the points are irregular.

λ.	L/D.	k_c.	$\delta - k_c$.	$L/W = \frac{\gamma}{\delta - k_c}$.	$\frac{a'}{\lambda}$.	$\frac{c'}{L/D}$.	$\theta = \frac{a'}{\lambda} - \frac{c'}{L/D}$.	$\frac{b'}{\lambda}$.	$\frac{d'}{L/D}$.	$\phi = \frac{b'}{\lambda} + \frac{d'}{L/D}$.	$V = \sqrt{a\theta L/W}$.	$P = \frac{\phi V^3}{375\theta}$.
·1	6·42	·650	1·650	1·212	9·568	·142	9·426	·510	·326	·836	179·7	1372
·2	11·41	·450	1·850	1·081	4·784	·080	4·704	·255	·183	·438	119·0	419
·3	15·65	·377	1·923	1·040	3·189	·058	3·131	·170	·133	·303	95·2	223
·4	17·63	·341	1·959	1·021	2·392	·052	2·340	·127	·118	·245	81·6	152
·5	16·15	·319	1·981	1·010	1·914	·057	1·857	·102	·129	·231	72·2	125
·6	13·81	·302	1·998	1·001	1·595	·066	1·529	·085	·151	·236	65·2	114
·7	13·03	·291	2·009	·996	1·367	·070	1·297	·073	·160	·237	60·0	105
·8	11·33	·287	2·013	·994	1·196	·081	1·115	·064	·184	·248	55·6	102
·9	9·64	·280	2·020	·991	1·063	·095	·968	·057	·217	·274	51·7	104
1·0	8·05	·276	2·024	·988	·957	·113	·844	·051	·240	·291	48·2	103

Example (4).—*This Example Shows the Fourth Method.*—

Suppose $W = 5700$, $S = 695$, $S' = 95$, $k_{L max} = \cdot 590$, $R_1 = 135$, $R_2 = 375$, $c = 7$, $l = 14$, $l' = 16 \cdot 1$, $d = 150$, $h_1 = 2 \cdot 5$, $h_2 = 1$,

$$\therefore\ a' = 1 - \frac{7 \cdot 2 \times 135}{150^2} = 1 - \cdot 0432 = \cdot 9568,$$

$$b' = \frac{510}{10^4} = \cdot 051,$$

$$c' = \frac{367 \times \cdot 590 \times \cdot 95}{150^2} = \cdot 914,$$

$$d' = \cdot 0051 \times \cdot 590 \times 695 = 2 \cdot 09,$$

$$\alpha = 5700 \times 14 = 79800,$$

$$\beta = \cdot 9568 \times 2 \cdot 09 + \cdot 051 \times \cdot 914 = 2 \cdot 00 + \cdot 047 = 2 \cdot 047,$$

$$A' = 2 \cdot 047 \times 16 \cdot 1 = 32 \cdot 93,$$

$$B' = 2 \cdot 047 \times 7 = 14 \cdot 32,$$

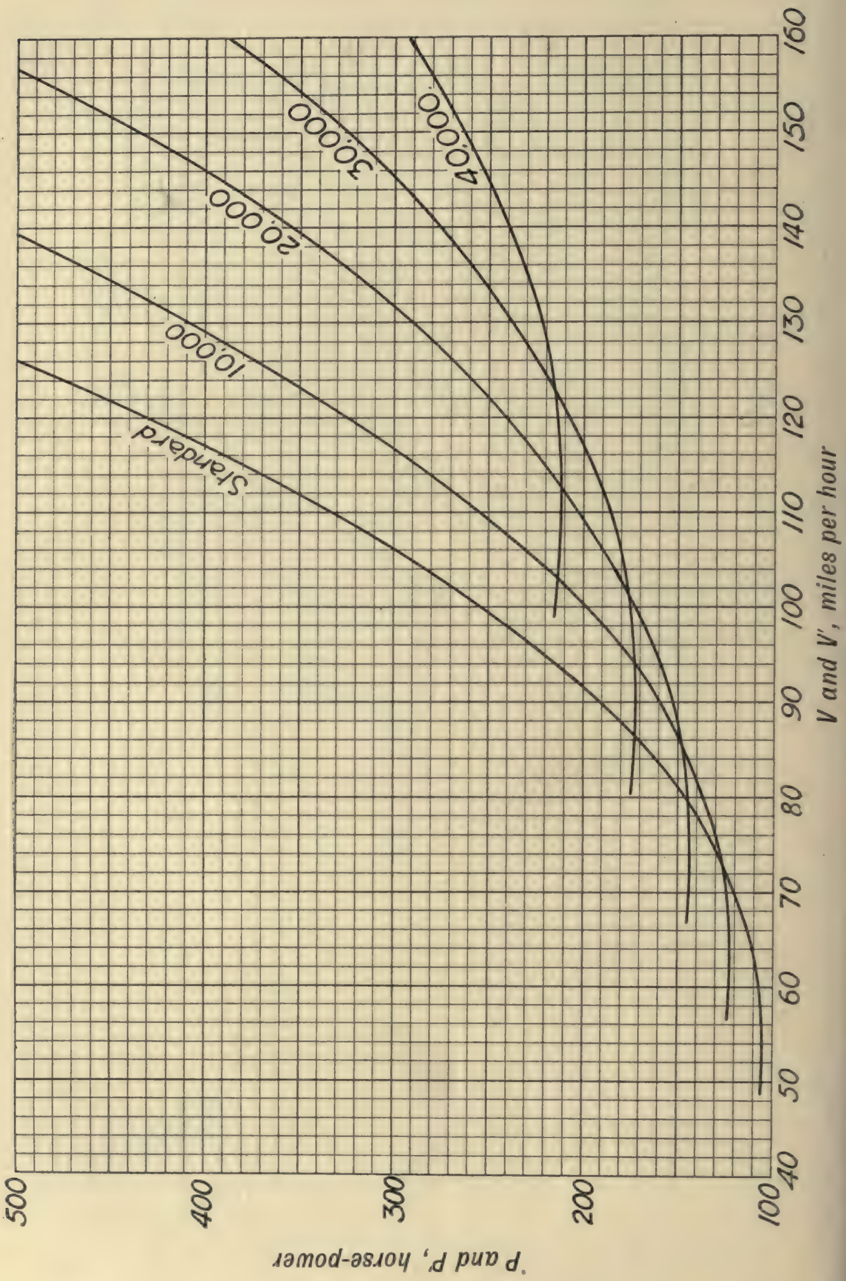
$$C' = 2 \cdot 5 \times \cdot 051 = \cdot 1275,$$

$$D' = 2 \cdot 5 \times 2 \cdot 09 - 1 \cdot 5 \times 2 \cdot 047, = 5 \cdot 225 - 3 \cdot 070 = 2 \cdot 155.$$

Now see table on next page.

The values of P and V are plotted on page 157, and the curve of short dashes is drawn through them, the opportunity being taken to fair the curve slightly where the points are irregular. For most of its length this curve is indistinguishable from the chain dotted curve.

λ.	L/D.	k_c.	$\frac{a'}{\lambda}$.	$\frac{c'}{L/D}$.	$\theta=\frac{a'}{\lambda}-\frac{c'}{L/D}$.	$\frac{b'}{\lambda}$.	$\frac{d'}{L/D}$.	$\phi=\frac{b'}{\lambda}+\frac{d'}{L/D}$.	$B'k_c$.	$\frac{C'}{\lambda}$.	$\frac{D'}{L/D}$.	$\gamma=B'k_c+\frac{C'}{\lambda}+\frac{D'}{L/D}$.	$\psi=A'-\gamma$.	$V=\sqrt{\frac{\alpha\theta}{\psi}}$.	$P=\frac{\phi V^3}{375\theta}$.
·1	6·42	·650	9·568	·142	9·426	·510	·326	·836	9·310	1·275	·336	10·921	22·01	184·8	1492
·2	11·41	·450	4·784	·080	4·704	·255	·183	·438	6·445	·637	·189	7·271	25·66	121·0	440
·3	15·65	·377	3·189	·058	3·131	·170	·133	·303	5·400	·425	·138	5·963	26·97	96·4	231
·4	17·63	·341	2·392	·052	2·340	·127	·118	·245	4·884	·319	·122	5·325	27·61	82·3	155
·5	16·15	·319	1·914	·057	1·857	·102	·129	·231	4·570	·255	·133	4·958	27·97	72·8	128
·6	13·81	·302	1·595	·066	1·529	·085	·151	·236	4·327	·212	·156	4·695	28·24	65·8	117
·7	13·03	·291	1·367	·070	1·297	·073	·160	·237	4·170	·182	·165	4·517	28·41	60·4	107
·8	11·33	·287	1·196	·081	1·115	·064	·184	·248	4·110	·159	·190	4·459	28·47	55·9	104
·9	9·64	·280	1·063	·095	·968	·057	·217	·274	4·010	·142	·224	4·376	28·55	52·0	106
1·0	8·05	·276	·957	·113	·844	·051	·240	·291	3·953	·127	·268	4·348	28·58	48·6	106


Standard
10,000
20,000
30,000
40,000
V and V', miles per hour
40
50
60
70
80
90
100
110
120
130
140
150
160
P and P', horse-power
100
200
300
400
500

Example (5).—*Given the machine performance curve in standard density air, find the machine performance curve at* 10,000, 20,000, 30,000, *and* 40,000 *feet.*

The values of σ at these altitudes are ·740, ·530, ·368, and ·242.

Altitude.		10,000.		20,000.		30,000.		40,000.	
V.	P.	V′.	P′.	V′.	P′.	V′.	P′.	V′.	P′.
184·8	1492	215·0	1735	254·0	2050	304·9	2460	375·9	3033
121·0	440	140·6	512	166·2	604	199·5	726	246·0	894
96·4	231	112·0	268	132·4	317	158·9	381	196·0	469
82·3	155	95·8	180	113·0	213	135·5	256	167·2	315
72·8	128	84·6	149	100·0	176	120·0	211	148·0	260
65·8	117	76·5	136	90·4	161	108·5	193	133·7	238
60·4	107	70·2	124	83·0	147	99·6	176	122·7	217
55·9	104	65·0	121	76·8	143	92·2	171	113·5	211
52·0	106	60·4	123	71·4	145	85·7	175	105·6	215
48·6	106	56·5	123	66·8	145	80·2	175	98·8	215

The five curves are all plotted on page 162.

CHAPTER XIX.

AIR PERFORMANCE.

I. GLIDING FLIGHT.

Example (1).—*Landing Speed on Glide: First Method.*—

Applying this method to the machine dealt with in examples (1) to (4), Chapter XVIII., page 155, we have—

$$W = 5700,$$
$$S = 695,$$
$$k_{L max} = \cdot 590,$$
$$\therefore\ V = \sqrt{\frac{196 \times 5700}{695 \times \cdot 590}} = 52\cdot 2 \text{ miles per hour.}$$

While at an altitude of, say, 15,000 feet, where $\sigma = \cdot 630$,

$$V' = \sqrt{\frac{196 \times 5700}{\cdot 630 \times 695 \times \cdot 590}} = 65\cdot 8 \text{ miles per hour.}$$

Example (2).—*Landing Speed on Glide: Second Method.*—

Again with the same machine—

$$W = 5700,$$
$$S = 695,$$
$$k_{L max} = \cdot 590,$$
$$l = 14,$$
$$l' = 16\cdot 1,$$
$$c = 7,$$
$$k_c = \cdot 276 \text{ (for } \lambda = 1\cdot 0),$$
$$\therefore\ V = \sqrt{\frac{196 \times 5700 \times 14}{695 \times \cdot 590(16\cdot 1 - 7 \times \cdot 276)}} = 51\cdot 8 \text{ miles per hour,}$$

and for the gliding landing speed in summer on a plateau in Mexico 6,000 feet above sea-level, allowing also for the relative air density correction (see footnote page 117) of 4,450 feet, we have to take σ corresponding to 10,450 feet, *i.e.* $\sigma = \cdot 725$.

$$\therefore\ V' = \sqrt{\frac{196 \times 5700 \times 14}{\cdot 725 \times 695 \times \cdot 590(16\cdot 1 - 7 \times \cdot 276)}}$$
$$= 60\cdot 9 \text{ miles per hour.}$$

Example (3).—*Gliding Angle.* —Taking the same machine again we have—

$$R = 510,$$

$$k_{L_{max}} = \cdot590, \qquad a = \frac{510}{51 \times \cdot590 \times 695} = \cdot02440,$$

$$S = 695.$$

λ.	L/D.	$\frac{a}{\lambda}$.	$\frac{1}{L/D}$.	$\tan\theta = \frac{a}{\lambda} + \frac{1}{L/D}$.
·1	6·42	·2440	·1558	·3998
·2	11·41	·1220	·0876	·2096
·3	15·65	·0813	·0639	·1452
·4	17·63	·0610	·0567	·1177
·5	16·15	·0488	·0619	·1107
·6	13·81	·0407	·0724	·1131
·7	13·03	·0349	·0768	·1117
·8	11·33	·0305	·0883	·1188
·9	9·64	·0271	·1038	·1309
1·0	8·05	·0244	·1243	·1487

The value of tan θ (and also the value of V taken from example (1), Chapter XVIII., page 155) is plotted on λ as a base on page 167.

From this curve we get that the minimum value of tan θ is ·1107 and occurs at a speed of about 74 miles per hour.

Therefore the Air Speed Indicator reading for best glide in still air at any altitude is 74, and the tangent of the best gliding angle in still air at any altitude is ·1107.

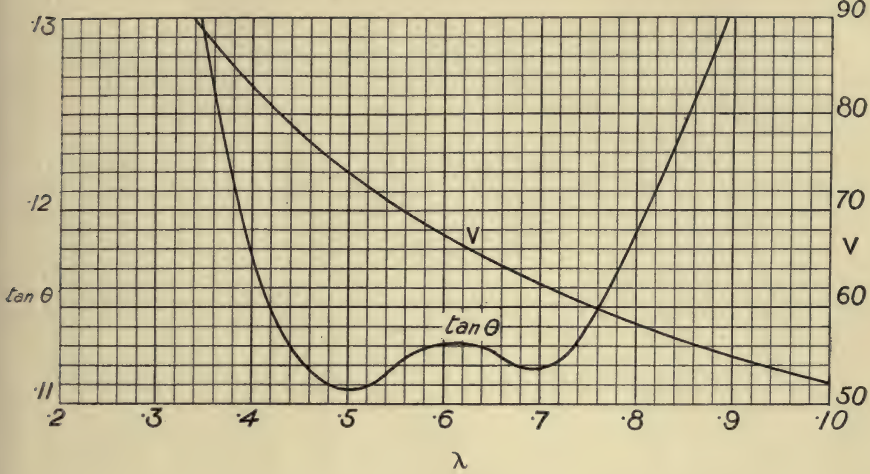
Example (4).—*Gliding in a Wind.*—Taking once again the same machine and taking as the wind speeds to consider—

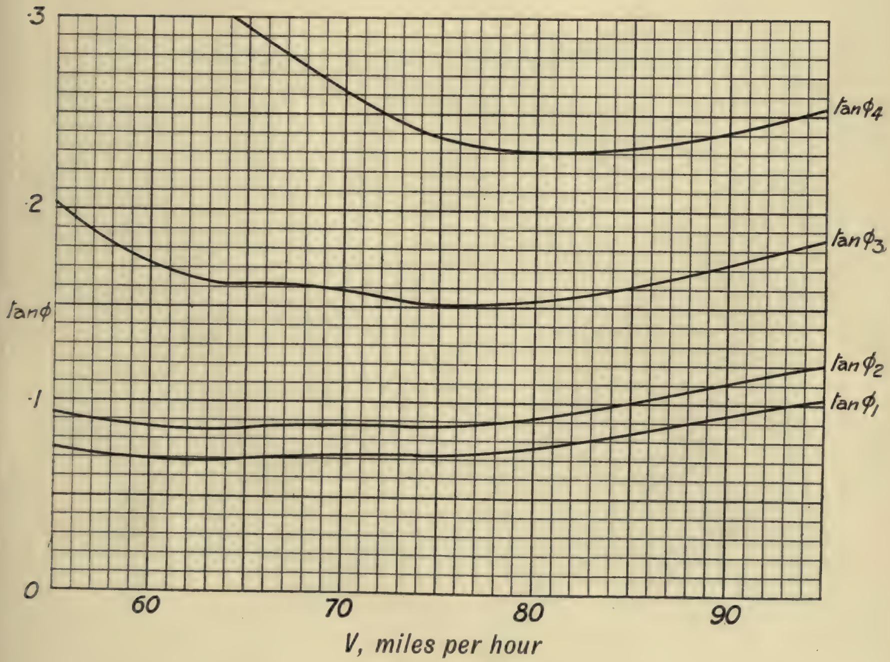
$$v_1 = -40 \text{ miles per hour},$$
$$v_2 = -20,$$
$$v_3 = 20,$$
$$v_4 = 40,$$

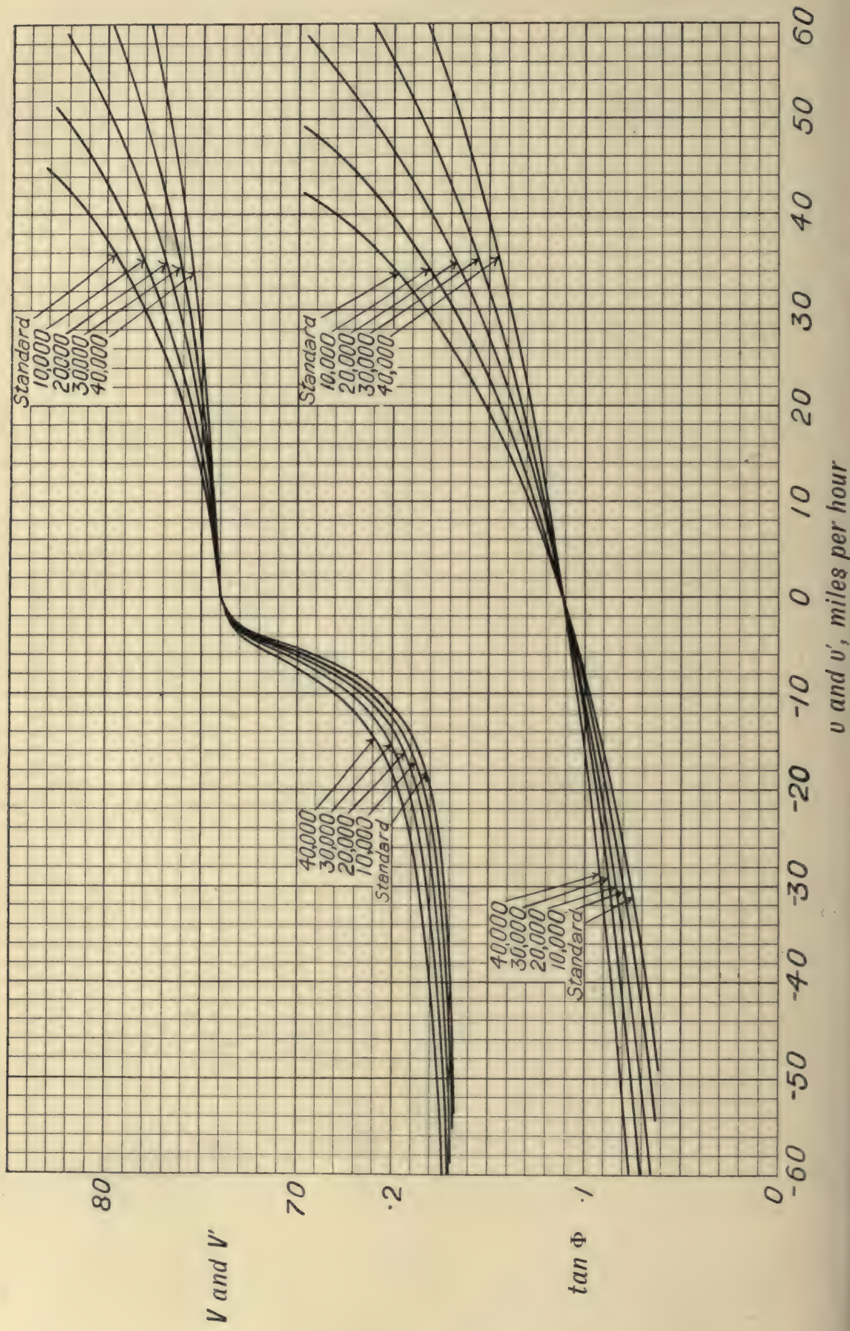
we have—

$$W = 5700,$$
$$R = 510,$$
$$a = \cdot02440.$$

λ.	L/D.	$\tan\theta$.	$\frac{\tan\theta}{L/D}$.	$1 + \frac{\tan\theta}{L/D}$.	$\frac{\lambda}{\alpha}\left(1 + \frac{\tan\theta}{L/D}\right)$.	$\frac{\lambda}{\alpha}\left(1 + \frac{\tan\theta}{L/D}\right) + \tan\theta$.	$V = \sqrt{\frac{10^4 W}{R\left[\frac{\lambda}{\alpha}\left(1 + \frac{\tan\theta}{L/D}\right) + \tan\theta\right]}}$.	$\frac{v_1}{V}$.	$1 - \frac{v_1}{V}$.	$\tan\phi_1 = \frac{\tan\theta}{1 - \frac{v_1}{V}}$.	$\frac{v_2}{V}$.	$1 - \frac{v_2}{V}$.	$\tan\phi_2 = \frac{\tan\theta}{1 - \frac{v_2}{V}}$.	$\frac{v_3}{V}$.	$1 - \frac{v_3}{V}$.	$\tan\phi_3 = \frac{\tan\theta}{1 - \frac{v_3}{V}}$.	$\frac{v_4}{V}$.	$1 - \frac{v_4}{V}$.	$\tan\phi_4 = \frac{\tan\theta}{1 - \frac{v_4}{V}}$.
·1	6·42	·3998	·062	1·062	4·355	4·755	153·2	− ·261	1·261	·3170	− ·130	1·130	·3535	·130	·870	·4595	·261	·739	·5410
·2	11·41	·2096	·018	1·018	8·360	8·570	113·5	− ·352	1·352	·1550	− ·176	1·176	·1781	·176	·824	·2544	·352	·648	·3235
·3	15·65	·1452	·009	1·009	12·40	12·55	94·4	− ·424	1·424	·1020	− ·212	1·212	·1198	·212	·788	·1844	·424	·576	·2522
·4	17·63	·1177	·007	1·007	16·50	16·62	82·0	− ·488	1·488	·0792	− ·244	1·244	·0948	·244	·756	·1558	·488	·512	·2300
·5	16·15	·1107	·007	1·007	20·64	20·75	73·4	− ·545	1·545	·0717	− ·272	1·272	·0871	·272	·728	·1522	·545	·455	·2435
·6	13·81	·1131	·008	1·008	24·79	24·90	67·0	− ·597	1·597	·0708	− ·299	1·299	·0872	·299	·701	·1614	·597	·403	·2809
·7	13·03	·1117	·009	1·009	28·91	29·02	62·1	− ·644	1·644	·0680	− ·322	1·322	·0846	·322	·678	·1648	·644	·356	·3140
·8	11·33	·1188	·010	1·010	33·10	33·22	58·0	− ·690	1·690	·0704	− ·345	1·345	·0884	·345	·655	·1815	·690	·310	·3836
·9	9·64	·1309	·014	1·014	37·40	37·53	54·6	− ·732	1·732	·0756	− ·366	1·366	·0958	·366	·634	·2066	·732	·268	·4888
1·0	8·05	·1487	·018	1·018	41·70	41·85	51·7	− ·774	1·774	·0839	− ·387	1·387	·1072	·387	·613	·2430	·774	·226	·6588


·13
·12
tan θ
·11
90
80
70
V
60
50
V
tan θ
·2
·3
·4
·5
·6
·7
·8
·9
·10
λ


·3
·2
tan φ
·1
0
tan φ4
tan φ3
tan φ2
tan φ1
60
70
80
90
V, miles per hour


Standard
10,000
20,000
30,000
40,000
V and V'
tan Φ
v and v', miles per hour
80
70
·2
·1
0
-60
-50
-40
-30
-20
-10
0
10
20
30
40
50
60

Altitude in Feet.			10,000.	20,000.	30,000.	40,000.
V.	v.	tan Φ.	v'.	v'.	v'.	v'.
62	−40	·068	−46·5	−55·0	−66·0	−81·4
63	−20	·085	−23·2	−27·5	−33·0	−40·7
74	0	·111	0	0	0	0
76	20	·151	23·2	27·5	33·0	40·7
81	40	·230	46·5	55·0	66·0	81·4

In this table the values of V and tan Φ for $v = 0$ are filled in from the immediately preceding piece of work: the remainder from the minima of the curves plotted on page 167, these curves being plotted from the data in the big table on page 166.

The values of σ for the altitudes in the table are ·740, ·530, ·368, and ·242. The last four columns are filled up from the formula $v' = \dfrac{v}{\sqrt{\sigma}}$.

V and tan Φ are then plotted on a wind speed base on page 168.

If desired they can be cross plotted on an altitude base by taking vertical sections at even intervals of wind speed.

II. Full Power Flight.

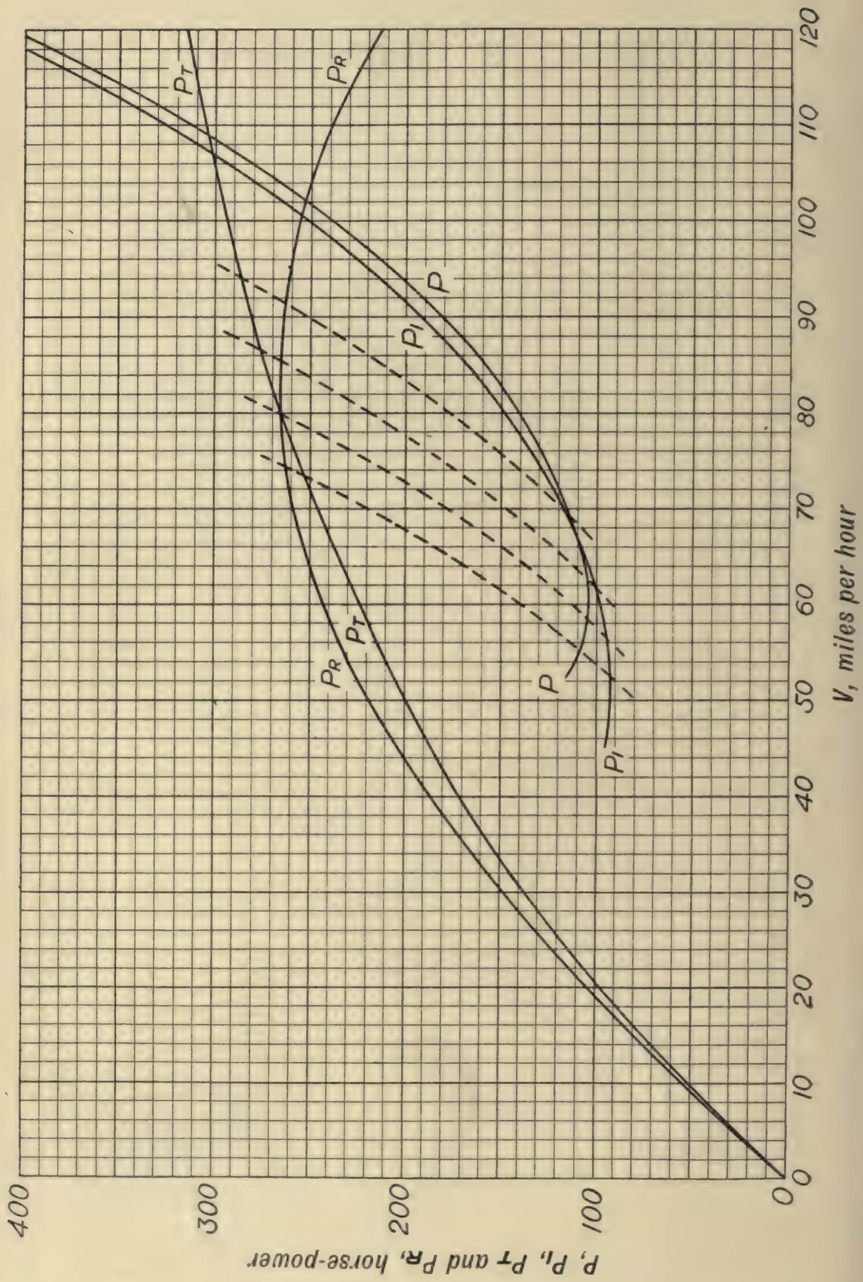
Example (5).—*Top Speed.*—Take the curves on page 170. The P curve is taken from example (1), Chapter XVIII., page 155, and the P_T and P_R curves are taken from example (3), Chapter XVII., page 153.

In this case the intersection of the P curve with the P_R curve is lower than the intersection of the P curve with the P_T curve. The first-mentioned intersection must therefore be taken: it gives—

top speed = 102 miles per hour.

For top speed at an altitude the procedure is the same, but the machine and propeller performance curves must be plotted for the altitude.

Example (6).—*Rate of Climb* (*First Approximation*).—Using the same curves as in example (5) and noting that W = 5700 for the machine they refer to, if we only require the maximum


V, miles per hour
0
10
20
30
40
50
60
70
80
90
100
110
120
P, P_I, P_T and P_R, horse-power
0
100
200
300
400
P
P_I
P_T
P_R

value, we observe that it occurs where $(P_p - P)$ is a maximum. A measurement of the vertical distance between the P_p and P curves shows at once that the maximum is at V = 71.

$$\therefore\ C_{max} = 33{,}000\,\frac{248 - 117}{5700} = 758 \text{ feet per minute.}$$

Also this maximum rate of climb occurs at a speed of 71 miles per hour.

Example (7).—*Rate of Climb* (*Second Approximation*).—Taking the same data as in example (6), and taking points at values of V of 55, 60, 70, 80, and 90:—

V.	P_p.	P.	P_p+P.	P_p-P.	$\frac{WV}{375(P_p+P)}$.	$\left\{\frac{WV}{375(P_p+P)}\right\}^2$.	$\frac{P_p-P}{P_p+P}$.	$\left\{\frac{WV}{375(P_p+P)}\right\}^2 - \frac{P_p-P}{P_p+P}$.	$\sqrt{\left\{\frac{WV}{375(P_p+P)}\right\}^2 - \frac{P_p-P}{P_p+P}}$.	$x=\frac{WV}{375(P_p+P)} - \sqrt{\left\{\frac{WV}{375(P_p+P)}\right\}^2 - \frac{P_p-P}{P_p+P}}$.	$1+x^2$.	$C=\frac{176Vx}{1+x^2}$.
55	212	109	321	103	2·603	6·780	·321	6·459	2·541	·062	1·004	598
60	219	104	323	115	2·823	7·974	·356	7·618	2·760	·063	1·004	662
70	246	116	362	130	2·940	8·646	·359	8·287	2·880	·060	1·004	736
80	266	141	407	125	2·990	8·942	·307	8·635	2·939	·051	1·003	716
90	264	182	446	82	3·069	9·420	·184	9·236	3·039	·030	1·001	474

The plotting of C on V (which there is no need to reproduce here) gives—

$$C_{max} = 744 \text{ feet per minute.}$$

Also this maximum rate of climb occurs at a speed of 74 miles per hour.

In using this method it must be remembered that even a twenty inch slide rule hardly gives the necessary accuracy. Therefore if the x column comes out with too few significant figures the work should be done with a table of logarithms.

Example (8).—*Rate of Climb (Third Approximation).*—Considering again the same machine as before—

$S' = 85$ (for the 135 inch propeller).

$k_{Lmax} = \cdot 590.$

$d = 135.$ $\qquad a = \dfrac{137{,}500 \times 85 \times \cdot 590}{135^2} = 378 \cdot 2$

$W = 5700.$

$R_1 = 135.$ $\qquad \beta = \dfrac{7 \cdot 2 \times 135}{135^2} = \cdot 0533.$

Now see table on next page.

The curve of P_1 on V_1 as a base is plotted on page 170.

Applying the method of the first approximation we find that the maximum value of $P_p - P_1$ occurs at a speed of 70 miles per hour.

$$\therefore\ C_{max} = 33{,}000 \frac{245 - 123}{5700} = 707 \text{ feet per minute.}$$

Also this maximum rate of climb occurs at a speed of 70 miles per hour.

Example (9).—*Times to Altitudes when the Curve of Rate of Climb on a Base of Altitude is not Approximately a Straight Line.*—

Suppose we have the following data :—

$C = 500$ at standard altitude (*i.e.* at 800 feet).

$C' = 385$, 193, and 50, at 4000, 8000, and 10,000 feet respectively.

Then we plot $\frac{1}{C}$ and $\frac{1}{C'}$ on an altitude base, and to get the time to, say, 7000 feet, we take the area under the curve from 0 to 7000 feet.

This area equals 18·4 in a unit which is the product of one foot of altitude and $\dfrac{1}{1 \text{ foot per minute climb}}$, *i.e.* the unit is minutes,

$\therefore$ the time to 7000 feet is 18·4 minutes.

Example (10).—*Times to Altitudes when the Curve of Rate of Climb is Approximately Straight.*—

Suppose we have the following data :—

$C = 500$ at 800 feet.

$C' = 370$, 193, and 100 respectively at 4000, 8000, and 10,000 feet.

λ.	L/D.	V.	P.	P_P.	$L = \frac{\alpha\lambda P_P}{V}$.	W − L.	$V_1 = V\sqrt{\frac{W-L}{W}}$.	βP_P.	$\frac{LV}{375 L/D}$.	$\gamma = \beta P_P + \frac{LV}{375 L/D}$.	P + γ.	$P_1 = (P+\gamma)\left(\frac{V_1}{V}\right)^3$.
·1	6·42	165·0	1002	—	—	—	—	—	—	—	—	—
·2	11·41	116·7	372	223	145	5555	115·2	12	4	16	388	374
·3	15·65	95·4	210	260	309	5391	92·8	14	5	19	229	211
·4	17·63	82·6	148	266	487	5213	79·1	14	6	20	168	148
·5	16·15	73·9	124	254	650	5050	69·6	14	8	22	146	122
·6	13·81	67·4	116	240	807	4893	62·4	13	10	23	139	110
·7	13·03	62·4	106	229	972	4728	56·2	12	12	24	130	95
·8	11·33	58·4	105	220	1140	4560	52·2	12	16	28	133	95
·9	9·64	55·0	109	212	1313	4387	48·3	11	20	31	140	95
1·0	8·05	52·2	118	205	1485	4215	44·9	11	26	37	155	99

A straight line which approximates well to these on the plotting of C and C′ on altitude gives $c = 535$ and $a_1 = 12{,}450$.

Then the time t in minutes to, say, 9000 feet is given by

$$t = \frac{2 \cdot 303 \times 12{,}450}{535} \log_{10}\left(\frac{12{,}450}{12{,}450 - 9000}\right) = 29 \cdot 9 \text{ minutes.}$$

The plottings in this and the previous example are not given as they are perfectly simple and ordinary.

Example (11).—*Ceiling.*—Taking the P and P_T curves of page 170 and laying the celluloid throttling curves over them we take four values each of V and V_T, using alternate throttling curves so as to cover a wide range. The throttling curves used are indicated in dotted curves on page 170.

Suppose we are dealing with a stationary engine, then $p = \cdot 161$ and $q = \cdot 839$.

V.	V_T.	$\sigma_1 = \left(\frac{V}{V_T}\right)^2$.	$q\sigma_1$.	$\sigma = q\sigma_1 + p$.	Altitude.
55·1	73·4	·564	·473	·534	14,800
58·3	79·9	·533	·447	·608	16,000
62·7	86·5	·526	·441	·602	16,400
69·3	94·0	·544	·456	·617	15,700

The plotting of Altitude on V (which there is no need to give here) gives the maximum ceiling as 16,410 feet at an Air Speed Indicator Reading of 62.

III. Throttled Flight.

Example (12).—*Slowest Flying Speed.*—Consider the machine performance curves plotted on page 157: curve (4) has been obtained by the Fourth Method, and therefore the slowest flying speed can be read off it directly: it is 48½ miles per hour.

Example (13).—*Consumption and Revolutions when Throttled.*—Take the case of standard density air, and work to the P, P_T, and P_R curves plotted on page 170. A certain number of throttling curves are shown on the plotting: using these we proceed, for a stationary engine, for which $p = \cdot 161$:—

V.	V_T.	V_R.	$(1-p)\left(\frac{V}{V_T}\right)^2$.	$p+(1-p)\left(\frac{V}{V_T}\right)^2$.	$\alpha=\left[p+(1-p)\left(\frac{V}{V_T}\right)^2\right]\frac{V}{V_R}$.	$\beta=\frac{V}{V_R}$.
55·1	73·4	74·3	·473	·634	·470	·742
58·3	79·9	79·9	·447	·608	·443	·730
62·7	86·5	85·3	·441	·602	·443	·735
69·3	94·0	91·4	·456	·617	·467	·758

Other Examples.—The remaining methods of Chapter XII., page 116, are most conveniently exemplified in the course of the complete set of calculations for a typical machine which are given in Chapter XXII., page 182. For examples on Best Cruising Speed, Best Cruising Altitude, and Cruising Range, therefore, see Chapter XXII., pages 197 and 204.

CHAPTER XX.

GROUND PERFORMANCE.

Example (1).—*Getting off a Deck.*—Take the case of the machine dealt with in Chapter XVIII., examples (1), (2), (3), and (4), page 155, with the propeller dealt with in Chapter XVII., example (3), page 153. The machine performance curves are plotted on page 157, and from curve (1) we estimate as a reasonable guess V = 37.

Now turn to the propeller performance curve which is plotted on page 170, from which we got $P_T = 162$.

$$\therefore\ T = 12{,}070\ \frac{162}{37} = 52{,}860.$$

Now suppose that $\lambda = \cdot 4$ is chosen for the run along the deck, then L/D = 17·63,

$$\therefore\ a = 11\cdot 4 \times \cdot 4 \times \cdot 590 \times 52{,}860 = 142{,}300$$

$$\therefore\ \mathrm{K} = \frac{\cdot 001495 \times 142{,}300 \times 695 \times 135}{5700 \times 150^2 - 142{,}300 \times 95} + \cdot 001495 \times 510$$

$$+ \frac{\cdot 07625 \times \cdot 4 \times \cdot 590 \times 695 \times 5700 \times 150^2}{17\cdot 63(5700 \times 150^2 - 142{,}300 \times 95)} = 1\cdot 729$$

$$B = \frac{1{,}256 \times 52{,}860}{150^2 \times 37^2} = 2\cdot 10$$

$$b = \sqrt{2\cdot 10 + 1} - 1 = \cdot 760.$$

Now we turn to the figures of page 177 where the point A is found in the upper figure and also the inclination of the resultant to the wings: this inclination is 11°, giving $k_L' = \cdot 536$.

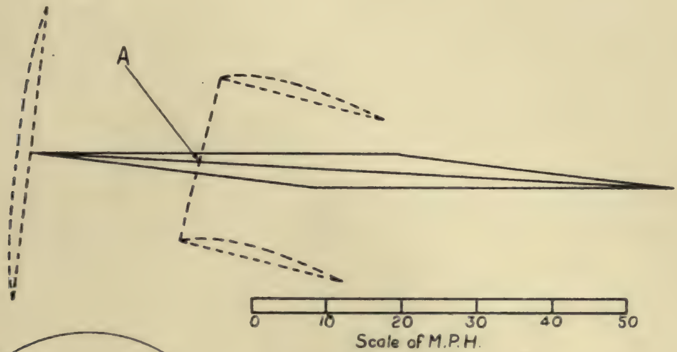
From the lower figure, the total length of leading edges is 104 feet, while the total length of leading edges in the circle is 18·5 feet.

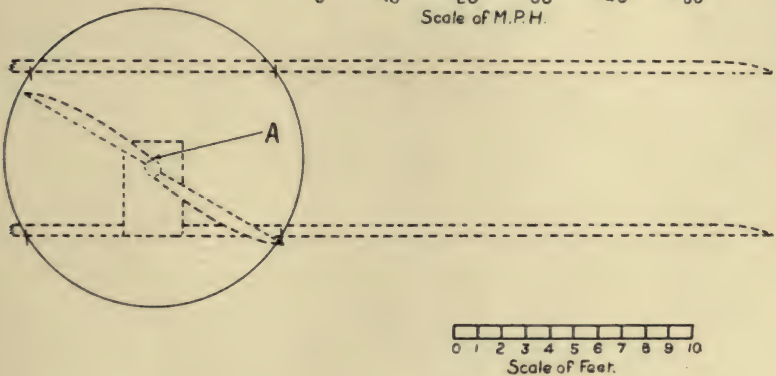
$$\therefore\ S'' = \frac{18\cdot 5 \times 695}{104} = 123\cdot 6 \text{ square feet.}$$

$$\therefore\ v_m = \sqrt{\frac{422 \times 5700}{\cdot 590(695 - 123\cdot 6) + \cdot 536(1\cdot 760)^2 123\cdot 6}} = 66\cdot 6$$

$$\beta = \sqrt{\frac{52,860}{1 \cdot 729}} = 175 \cdot 0.$$

Therefore the length run to get off in feet, supposing the ship is doing 32 miles per hour in a calm, so that $V_w = 32$ and $v_w = 1 \cdot 467 \times 32 = 46 \cdot 9$, is





$$\frac{1 \cdot 151 \times 5700}{1 \cdot 729}\left[\left(1 + \frac{46 \cdot 9}{175 \cdot 0}\right) \log_{10}\left(\frac{175 \cdot 0 + 46 \cdot 9}{175 \cdot 0 + 66 \cdot 6}\right) + \left(1 - \frac{46 \cdot 9}{175 \cdot 0}\right) \log_{10}\left(\frac{175 \cdot 0 - 46 \cdot 9}{175 \cdot 0 - 66 \cdot 6}\right)\right],$$

which equals $23 \cdot 7$ feet run measured along the deck.

Example (2).—*Getting off the Ground.*—For the same machine, the length run in still air in feet is

$$\frac{1{\cdot}151 \times 5700}{1{\cdot}729}\left[\log_{10}\left(\frac{175{\cdot}0}{175{\cdot}0 + 66{\cdot}6}\right) + \log_{10}\left(\frac{175{\cdot}0}{175{\cdot}0 - 66{\cdot}6}\right)\right]$$
$$= 258 \text{ feet.}$$

Example (3).—*Landing on a Deck.*—With the same ship speed and the same machine, $R_1 + R_2 = 510$, $k_{L max} = {\cdot}590$, $S = 695$, $L/D = 8{\cdot}05$ when $\lambda = 1{\cdot}0$.

$$\therefore K = {\cdot}001495 \times 510 + \frac{{\cdot}07625 \times {\cdot}590 \times 695}{8{\cdot}05} = 4{\cdot}646.$$

The slowest flying speed has been worked out for the machine we are dealing with in example (12), Chapter XIX., Part III., page 174.

$$\therefore V_m = 48{\cdot}5.$$

Also as before $V_w = 32{\cdot}0$. Therefore the length run on alighting, measured in feet along the deck is

$$\frac{5700}{4{\cdot}646}\left[2{\cdot}303 \log_{10}\left(\frac{48{\cdot}5}{32{\cdot}0}\right) - \frac{48{\cdot}5 - 32{\cdot}0}{48{\cdot}5}\right] = 93{\cdot}2 \text{ feet.}$$

Example (4).—*Landing on the Ground.*—Take the same machine landing on the ground in a calm, and assume that brakes on the wheels (if any) and tail skid friction on the ground are estimated by giving a value of ·04 to μ. Suppose that the machine is flown, not glided, down to the landing, then again $V_m = 48{\cdot}5$. As before $K = 4{\cdot}646$

$$\therefore a = \frac{4{\cdot}646}{5700} - \frac{14{\cdot}95 \times {\cdot}04}{48{\cdot}5^2} = {\cdot}000561.$$

Then the length of run in feet, for the value of μ assumed, is

$$\frac{1{\cdot}151}{{\cdot}000561} \log_{10}\left(\frac{4{\cdot}646 \times 48{\cdot}5^2}{14{\cdot}95 \times 5700 \times {\cdot}04}\right) = 1045 \text{ feet.}$$

CHAPTER XXI.

WATER PERFORMANCE.

Example.—*The Condition for Getting Off the Water.*—Suppose we are considering a flying boat whose hull lines and angular setting have been copied from a certain tank test, and whose performance curve is curve (4) plotted on page 157 and whose propeller curves are given on page 170.

Further suppose that the test made in the model tank has been issued in the following form :—

"Full sized machine, total weight 10,000 pounds, getting-off speed 46·875 knots : resistances in pounds for the full sized machine at 10, 15, 20, 23, 26, and 29 knots are 900, 1350, 1720, 1800, 1550, and 1300 respectively."

Our first care is to see if the necessary conditions of similarity are *all* fulfilled.

First, we must have geometrical similarity of the under water lines : we have already supposed that this condition is met.

Secondly, the hull must be set at the same angle relative to the wings in our machine as in the prototype—this condition also we have met.

Thirdly, we have to have

$$\left(\frac{\mathrm{W}}{W}\right) = \left(\frac{\mathrm{L}}{L}\right)^3$$

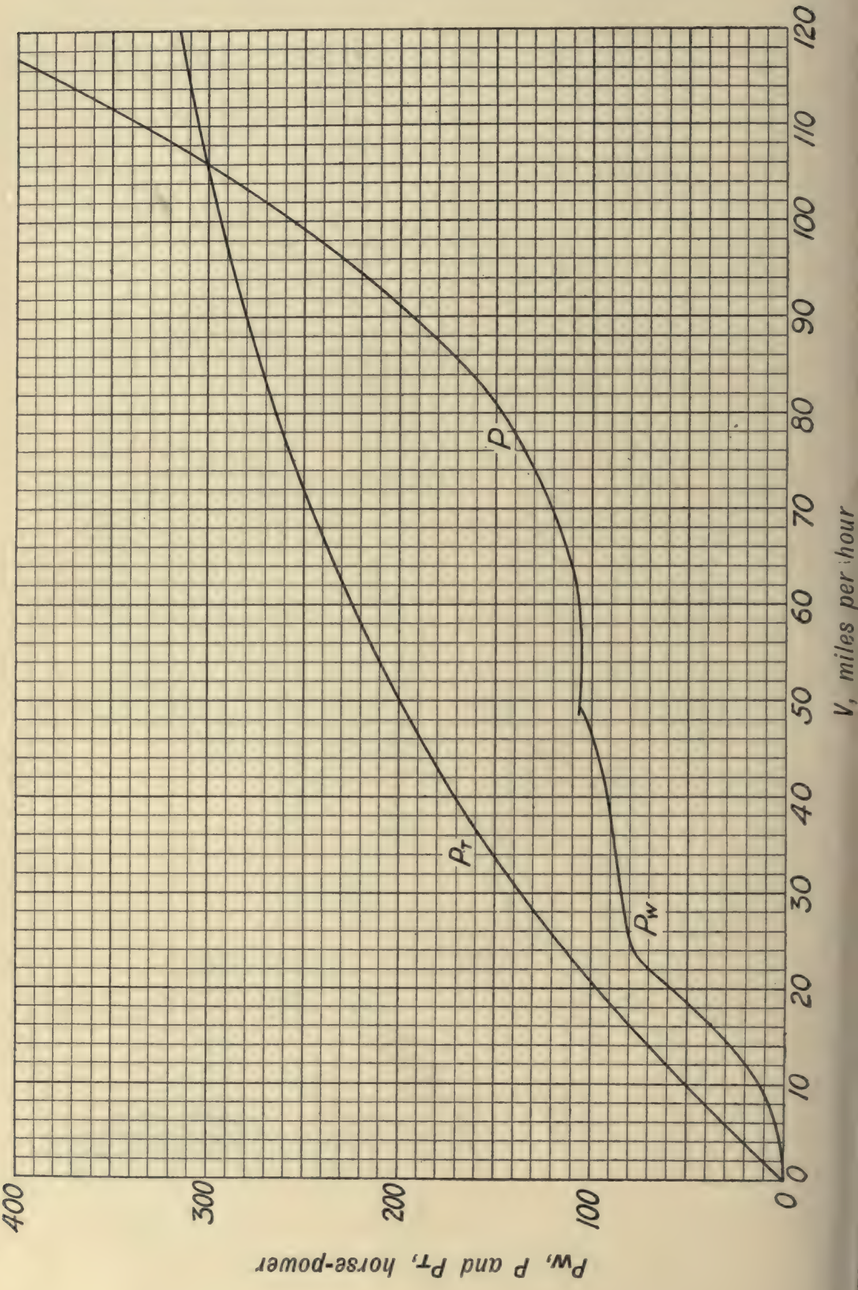
and we will suppose that in copying the old design we have been careful, of course, to meet this condition.

Fourthly, we have the condition to meet that our getting-off speed divided by that of the prototype equals

$$\sqrt{\frac{\mathrm{L}}{L}} = \left(\frac{\mathrm{W}}{W}\right)^{\frac{1}{6}}.$$

Now the prototype's getting-off speed is given as 46·875 knots = 46·875 × 1·151 miles per hour. Also we have W = 5700 and W = 10,000. Therefore our getting-off speed ought to be

$$46{\cdot}875 \times 1{\cdot}151\left(\frac{5700}{10{,}000}\right)^{\frac{1}{6}} = 49{\cdot}2 \text{ miles per hour.}$$


P
P_T
P_W
0
10
20
30
40
50
60
70
80
90
100
110
120
V, miles per hour
0
100
200
300
400
P_W, P and P_T, horse-power

Now actually our getting-off speed is 48·5 miles per hour, so we are quite near enough for practical purposes.

Now 10, 15, 20, 23, 26, and 29 knots are equivalent to 11·5, 17·3, 23·0, 26·5, 29·9, and 33·4 miles per hour. Therefore we have, observing that

$$\left(\frac{L}{\mathit{L}}\right)^3 = \frac{W}{\mathit{W}} = \frac{5700}{10{,}000} = \cdot 57 \text{ and } \sqrt{\frac{L}{\mathit{L}}} = (\cdot 57)^{\frac{1}{6}} = \cdot 91 :—$$

V.	*F.*	V.	F.
11·5	900	10·5	513
17·3	1350	15·7	770
23·0	1720	20·9	980
26·5	1800	24·1	1025
29·9	1550	27·2	884
33·4	1300	30·4	741

Now with V_1 = 49·2 miles per hour (using this figure so as to correspond correctly to the model tests) we obtain P_1 = 107; then :—

V.	F.	$P_1\left(\frac{V}{V_1}\right)^3$.	$\frac{VF}{375}$.	$P_w = P_1\left(\frac{V}{V_1}\right)^3 + \frac{VF}{375}$.
10·5	513	1	14	15
15·7	770	3	32	35
20·9	980	8	55	63
24·1	1025	13	66	79
27·2	884	18	64	82
30·4	741	25	60	85

P_w, P, and P_T are plotted on V on page 180, from which we see that the machine will get off the water in a calm.

CHAPTER XXII.

A TYPICAL MACHINE.

General.—This Chapter contains the complete performance calculations of a particular machine, as it is hoped by that means to elucidate points which may be obscure in the other parts of the book.

The machine we will deal with is illustrated by scale drawings on pages 184 and 185. The wing section is No. 64; the total weight is 6000 pounds made up as follows: structure including fuel tanks, engine mounting and cowling, instruments and all accessories, 2285 pounds; engine, 635 pounds; fuel (petrol and oil), 2400 pounds; cargo, 500 pounds; crew (pilot only), 180 pounds. The engine is a Dragonfly. As a designer will appreciate at once, such a machine is well adapted to be a long range carrier of special goods—on paper—but is open to serious criticisms for practical reasons: that, however, need not disturb us, as the machine is merely required to afford an example of a complete set of calculations. Other data about the machine will be given as they are required.

Body Resistance.—We will first decide on a propeller diameter. The machine is loaded roughly 10·7 pounds per square foot and 17·65 pounds per B.H.P. so that on general grounds we may anticipate a top speed of the order of 100 miles per hour if the propeller was designed for top speed.

The machine, however, is intended for a cruiser, and we will therefore suppose that the designer decides to have a 4-bladed propeller designed for 80 miles per hour. Then $V_0 = 80$ miles per hour and we get from page 99, taking $H = 340$ and $n_0 = 1750$,

$$d = 10{,}000\sqrt[4]{\frac{340}{111 \times 1750^2 \times 80}} = 106 \text{ inches.}$$

Hence we get the propeller circle shown dotted on the front-view drawing of page 185.

Parts in the Slip Stream.—From a consideration of the propeller circle on the front-view drawing we get—with a little give

and take—the following parts in the slip stream: fuselage, cockpit, windscreen, tail skid, fin and rudder, just half the tail plane and elevators, all tail bracing wires, all centre section gap struts, all chassis wires, half the chassis struts, all centre section bracing wires, all centre section incidence wires, all tail unit control levers, and all tail unit control wires.

We will now work out these items of resistance, putting a cross in the margin each time we arrive at a figure which will be required for later reference.

The *fuselage* has a maximum cross section consisting of 3 feet 0 inches × 4 feet 0 inches rectangular and a domed top 1 foot 3 inches high.

$$\therefore\ a = 12 + 2{\cdot}6 = 14{\cdot}6$$
$$\therefore\ r_1 = 2{\cdot}5 \times 14{\cdot}6 = 36{\cdot}5 \quad . \quad . \quad . \quad \text{(X)}$$

The *struts* on the list are 4 centre section gap struts 1 inch × 4 feet 0 inches, $\frac{1}{2}$ of 2 front chassis struts 2 inches × 2 feet 3 inches in front view, $\frac{1}{2}$ of 2 back chassis struts, $1\frac{3}{4}$ inches × 2 feet 3 inches in front view, and 6 king-levers on the tail 1 inch average × 1 foot 0 inches.

We will take it that all these struts are of R.A.F.4.z. section, *i.e.* section B, page 80, with fineness ratio 4.

$$\therefore\ a = 4 \times 1 \times 4 + \tfrac{1}{2} \times 2 \times 2 \times 2{\cdot}25 + \tfrac{1}{2} \times 2 \times 1{\cdot}75 \times 2{\cdot}25 + 6 \times 1 \times 1$$
$$= 16 + 4{\cdot}5 + 3{\cdot}94 + 6 = 30{\cdot}44\,;$$

and
$$x = {\cdot}215$$
$$\therefore\ r_1 = 30{\cdot}44 \times {\cdot}215 = 6{\cdot}5 \quad . \quad . \quad \text{(X)}$$

The *cockpit* is 1 foot 6 inches wide.

$$\therefore\ r_1 = {\cdot}7 \times 18 = 12{\cdot}6 \quad . \quad . \quad . \quad \text{(X)}$$

The "*normal plate*" items consist of the windscreen, 8 inches × 4 inches, and the tail skid, 6 inches × 2 inches.

$$\therefore\ a = \tfrac{2}{3} \times \tfrac{1}{3} + \tfrac{1}{2} \times \tfrac{1}{6} = {\cdot}2222 + {\cdot}0833 = {\cdot}3055$$
$$\therefore\ r_1 = 29 \times {\cdot}3055 = 8{\cdot}9 \quad . \quad . \quad . \quad . \quad . \quad \text{(X)}$$

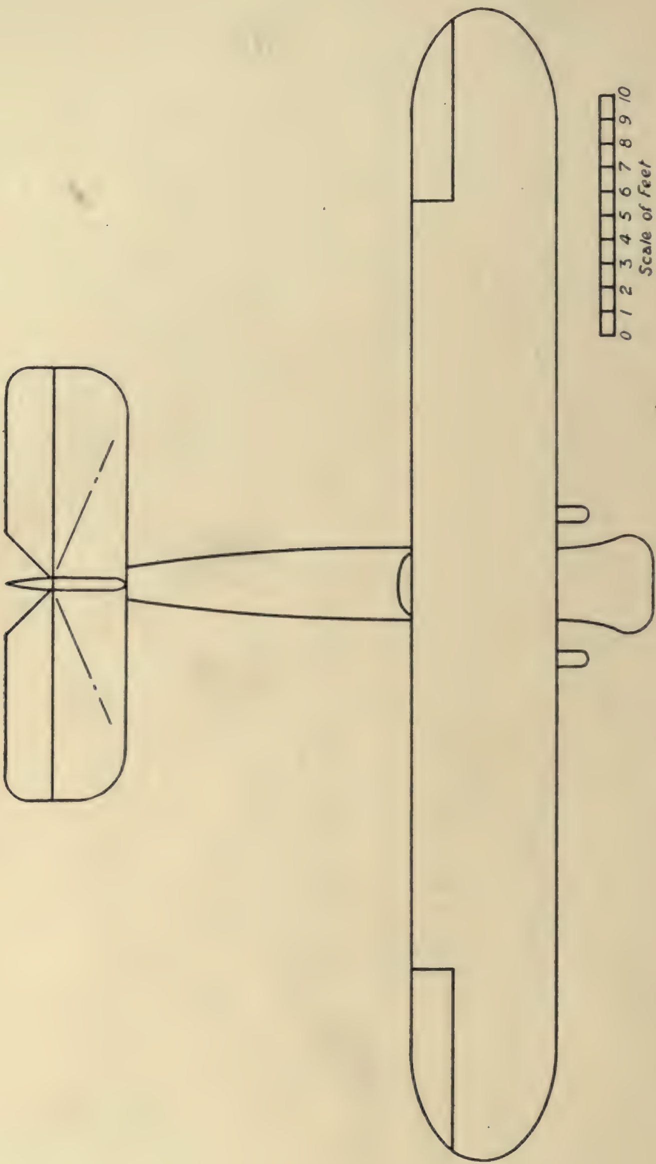
The *fin and rudder* area is 18·4 square feet.

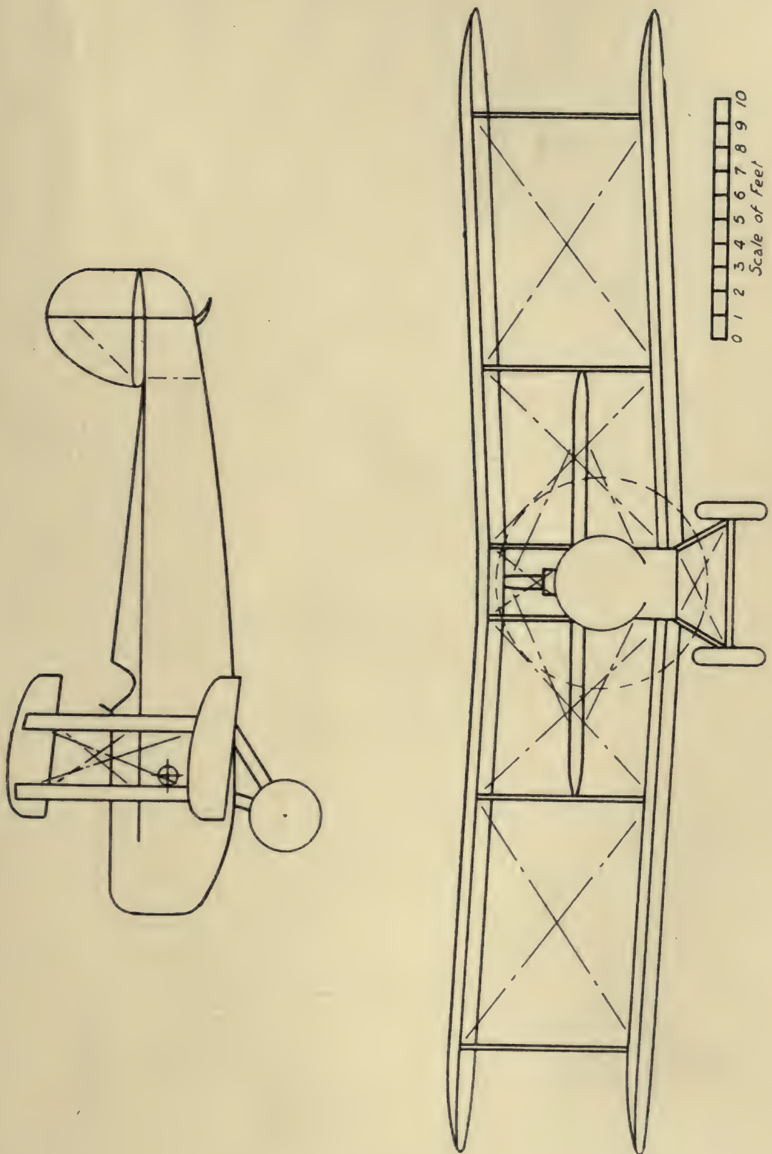
$$\therefore\ r_1 = {\cdot}58 \times 18{\cdot}4 = 10{\cdot}7 \quad . \quad . \quad . \quad \text{(X)}$$

The *tail plane and elevator* area is 83·4 square feet, and we are dealing with half of it.

$$\therefore\ r_1 = {\cdot}78 \times \frac{83{\cdot}4}{2} = 32{\cdot}5 \quad . \quad . \quad . \quad \text{(X)}$$

The *wires* in the list comprise 2 top front tail bracing wires $\frac{3}{8}$-inch stream-line 6 feet 0 inches long in front view, 6 other tail bracing wires $\frac{1}{4}$-inch stream-line 6 feet 0 inches long, 2 front chassis wires $\frac{11}{16}$-inch stream-line 4 feet 6 inches long, 2 back chassis wires $\frac{7}{16}$-inch stream-line 4 feet 6 inches long in front


0 1 2 3 4 5 6 7 8 9 10
Scale of Feet


0 1 2 3 4 5 6 7 8 9 10
Scale of Feet

view, 4 centre section bracing wires $\frac{1}{4}$-inch stream-line 4 feet 9 inches long, 4 centre section incidence wires $\frac{1}{2}$-inch stream-line 4 feet 0 inches long in front view, and 8 tail control wires 15 cwt. cable 3 feet 0 inches long in front view. Then observing that the diameter of 15 cwt. cable is ·137 inches we have—

$$\begin{aligned} r_1 = &\ {\cdot}025 \times {\cdot}375(72 + 250 \times {\cdot}375) \times 2 \\ &+ {\cdot}025 \times {\cdot}25(72 + 250 \times {\cdot}25) \times 6 \\ &+ {\cdot}025 \times {\cdot}688(54 + 250 \times {\cdot}688) \times 2 \\ &+ {\cdot}025 \times {\cdot}437(54 + 250 \times {\cdot}437) \times 2 \\ &+ {\cdot}025 \times {\cdot}25(57 + 250 \times {\cdot}25) \times 4 \\ &+ {\cdot}025 \times {\cdot}5(48 + 250 \times {\cdot}5) \times 4 \\ &+ {\cdot}26 \times {\cdot}137(36 + 200 \times {\cdot}137) \times 8 \\ &= 3{\cdot}11 + 5{\cdot}05 + 7{\cdot}78 + 3{\cdot}57 + 2{\cdot}99 + 8{\cdot}64 + 18{\cdot}05 = 49{\cdot}2 \quad . \quad (X) \end{aligned}$$

Hence we have

$$R_1 = 36{\cdot}5 + 6{\cdot}5 + 12{\cdot}6 + 8{\cdot}9 + 10{\cdot}7 + 32{\cdot}5 + 49{\cdot}2 = 157 \quad . \quad (X)$$

Parts Outside the Slip-Stream:—

The *tail plane*, or rather the half of it outside the slip-stream, gives

$$r_2 = 32{\cdot}5 \quad . \quad . \quad . \quad . \quad (X)$$

The *struts* outside the slip stream are 2 front outer gap struts $1\frac{1}{2}$ inches × 7 feet 6 inches, 2 back outer gap struts $1\frac{3}{8}$ inches × 7 feet 6 inches, 2 front inner gap struts $1\frac{7}{8}$ inches × 7 feet 6 inches, 2 back inner gap struts $1\frac{3}{4}$ inches × 7 feet 6 inches, $\frac{1}{2}$ of 2 front chassis struts 2 inches × 2 feet 3 inches, $\frac{1}{2}$ of 2 back chassis struts, $1\frac{3}{4}$ inches × 2 feet 3 inches, 1 axle $2\frac{5}{8}$ inches × 5 feet 6 inches, and 4 aileron king-levers 1 inch average × 1 foot 0 inches.

Taking all these struts also as R.A.F.4.z. section,

$$\begin{aligned} a = &\ 2 \times 1{\cdot}5 \times 7{\cdot}5 + 2 \times 1{\cdot}375 \times 7{\cdot}5 + 2 \times 1{\cdot}875 \times 7{\cdot}5 \\ &+ 2 \times 1{\cdot}75 \times 7{\cdot}5 + \tfrac{1}{2} \times 2 \times 2 \times 2{\cdot}25 + \tfrac{1}{2} \times 2 \times 1{\cdot}75 \times 2{\cdot}25 \\ &+ 1 \times 2{\cdot}625 \times 5{\cdot}5 + 4 \times 1 \times 1 \\ &= 22{\cdot}5 + 20{\cdot}6 + 28{\cdot}1 + 26{\cdot}3 + 4{\cdot}5 + 3{\cdot}9 + 14{\cdot}4 + 4{\cdot}0 \\ &= 124{\cdot}3 \end{aligned}$$

and $x = {\cdot}215$

$$\therefore\ r_2 = 124{\cdot}3 \times {\cdot}215 = 26{\cdot}7 \quad . \quad . \quad . \quad . \quad . \quad . \quad (X)$$

The *wires* outside the slip-stream are 4 outer incidence wires $\frac{1}{4}$-inch stream-line × 7 feet 6 inches, 4 inner incidence wires $\frac{3}{8}$-inch stream-line × 7 feet 6 inches, 2 front outer flying wires $\frac{3}{8}$-inch stream-line × 12 feet 4 inches, 2 back outer flying wires $\frac{5}{16}$-inch stream-line × 12 feet 4 inches, 2 front outer weight wires $\frac{1}{4}$-inch stream-line × 11 feet 7 inches, 2 back outer weight wires $\frac{1}{4}$-inch stream-line × 11 feet 7 inches, 2 front inner flying

wires $\frac{1}{2}$-inch stream-line × 10 feet 1 inch, 2 back inner flying wires $\frac{7}{16}$-inch stream-line × 10 feet 1 inch, 2 front inner weight wires $\frac{3}{8}$-inch stream-line × 9 feet 7 inches, 2 back inner weight wires $\frac{5}{16}$-inch stream-line × 9 feet 7 inches, and 4 aileron cables 15 cwt. × 1 foot 0 inches in front view.

$$\begin{aligned}
\therefore\ r_2 = {} & \cdot 025 \times \cdot 25(90 + 250 \times \cdot 25) \times 4 \\
& + \cdot 025 \times \cdot 375(90 + 250 \times \cdot 375) \times 4 \\
& + \cdot 025 \times \cdot 375(148 + 250 \times \cdot 375) \times 2 \\
& + \cdot 025 \times \cdot 312(148 + 250 \times \cdot 312) \times 2 \\
& + \cdot 025 \times \cdot 25(139 + 250 \times \cdot 25) \times 2 \\
& + \cdot 025 \times \cdot 25(139 + 250 \times \cdot 25) \times 2 \\
& + \cdot 025 \times \cdot 5(121 + 250 \times \cdot 5) \times 2 \\
& + \cdot 025 \times \cdot 437(121 + 250 \times \cdot 437) \times 2 \\
& + \cdot 025 \times \cdot 375(115 + 250 \times \cdot 375) \times 2 \\
& + \cdot 025 \times \cdot 312(115 + 250 \times \cdot 312) \times 2 \\
& + \cdot 26 \times \cdot 137(12 + 200 \times \cdot 137) \times 4 \\
& = 3\cdot 94 + 6\cdot 89 + 4\cdot 53 + 3\cdot 53 + 2\cdot 52 + 2\cdot 52 + 6\cdot 15 \\
& + 5\cdot 04 + 3\cdot 91 + 3\cdot 02 + 5\cdot 40 = 47\cdot 5 \quad . \quad . \quad . \quad (X)
\end{aligned}$$

The *wheels* for a machine of this weight will be 2 wheels, 900 × 200, and we will suppose them fully shielded.

$$\therefore\ r_2 = \cdot 000062 \times 900 \times 200 \times 2 = 22\cdot 3 \quad . \quad . \quad (X)$$

The "*flat plate*" list includes 1·5 square feet for the shock absorbers and chassis fittings and ·3 square feet for the ordinary wing fittings at the ends of the gap struts.

$$\therefore\ r_2 = 29 \times 1\cdot 8 = 52\cdot 2 \quad . \quad . \quad . \quad (X)$$

Hence we have, on adding up,

$$R_2 = 32\cdot 5 + 26\cdot 7 + 47\cdot 5 + 22\cdot 3 + 52\cdot 2 = 181 \quad . \quad (X)$$

and $\quad R_1 = 157 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (X)$

it being convenient to have R_1 and R_2 together for future reference.

Wing Characteristics.—Aspect Ratio $= \dfrac{48'\ 0''}{6'\ 0''} = 8 \quad (X)$

$$\text{Gap/chord} = \frac{7'\ 6''}{6'\ 0''} = 1\cdot 25 \quad . \quad . \quad . \quad (X)$$

$$\text{Stagger} = 6^\circ \quad . \quad . \quad . \quad . \quad . \quad . \quad (X)$$

$$\text{Wing tips} = \frac{4'\ 6''}{6'\ 0''} = \cdot 75 \quad . \quad . \quad . \quad (X)$$

Also if we take the data for No. 64 Wing Section from the tests quoted on page 87,

$$\text{Dimensions} = 3 \times 40 = 120 \quad . \quad . \quad . \quad (X)$$

λ.	Corrections for					Model L/D.	Corrected L/D.
	Aspect Ratio.	Gap / Chord.	Stagger.	Wing Tips.	Dimensions.		
·1	·940	1·064	1·017	1·000	1·084	5·75	6·33
·2	·964	·991	1·018	1·037	1·026	11·20	11·58
·3	·981	·946	1·012	1·045	1·048	15·50	15·93
·4	·988	·904	1·004	1·050	1·080	17·80	18·09
·5	1·111	·840	1·002	1·096	1·065	17·35	18·93
·6	1·170	·827	·996	1·062	1·016	16·20	16·85
·7	1·170	·815	1·005	1·071	1·031	14·80	15·65
·8	1·170	·794	1·005	1·070	1·009	13·50	13·60
·9	1·157	·785	1·005	1·037	1·006	12·20	11·61
1·0	1·111	·785	1·005	1·140	1·000	9·40	9·40
k_{Lmax}	1·003	·955	1·011	1·028	1·025	·617	·6295

The last column contains the data for use in later work.

Propeller Performance Curves.—We have already decided to use $V_0 = 80$, and found $d = 106$, also $n_0 = 1750$,

$$\therefore J = \frac{80}{1750 \times 106} = ·000431.$$

$\frac{V}{V_0}$.	η_T.	η_R.	V.	P_T.	P_R.
·4	·352	·370	32	120	126
·6	·487	·511	48	166	174
·8	·593	·614	64	202	209
1·0	·680	·680	80	231	231
1·2	·745	·700	96	253	238
1·4	·795	·674	112	270	229

The above, of course, is for standard density air.

Machine Performance Curve.—We will use the Variant of the Third Method. Then from data already quoted or found and from the drawing we obtain :—

$$W = 6000. \qquad a' = 1 - \frac{7{\cdot}2 \times 157}{106^2} = {\cdot}900.$$

$$S = 535. \qquad b' = \frac{157 + 181}{10^4} = {\cdot}0338.$$

$$S' = 47.$$

$$k_{L max} = {\cdot}6295. \qquad c' = \frac{367 \times {\cdot}6295 \times 47}{106^2} = {\cdot}992.$$

$$R_1 = 157. \qquad d' = {\cdot}0051 \times {\cdot}6295 \times 535 = 1{\cdot}716.$$

$$R_2 = 181.$$

$$d = 106. \qquad a = \frac{6000}{{\cdot}900 \times 1{\cdot}716 + {\cdot}0338 \times {\cdot}992} = 3805.$$

Now see table on next page.

The above, of course, is for standard density air.

Ceiling.—The curves for P_T, P_R, and P on V just determined are plotted on page 192, and the throttling curves are plotted across them (the *plotting* of the throttling curves is only necessary in order that they may appear in print: in actually making such a calculation as this, one only has to lay the celluloid curves over the squared paper, of course).

V.	V_T.	$\sigma_1 = \left(\frac{V}{V_T}\right)^2$.	$q\sigma_1$.	$\sigma = q\sigma_1 + p$.	Altitude.
55·2	66·1	·698	·586	·747	9,600
56·5	69·0	·671	·563	·724	10,700
57·9	72·0	·647	·543	·704	11,500
59·5	75·4	·623	·523	·684	12,500
61·1	78·5	·606	·508	·669	13,100
63·1	82·1	·591	·496	·657	13,700
65·3	85·5	·583	·489	·650	14,100
68·1	89·2	·583	·489	·650	14,100
71·2	92·8	·589	·494	·655	13,800
75·0	96·8	·601	·504	·665	13,300
79·6	100·8	·624	·523	·684	12,500
86·3	104·9	·677	·568	·729	10,500
100·0	109·0	·841	·705	·866	5,300

The engine being a stationary one, $q = {\cdot}839$, and $p = {\cdot}161$, of course.

Now V is the Air Speed Indicator reading for which the

λ.	L/D.	$\frac{a'}{\lambda}$.	$\frac{c'}{L/D}$.	$\theta = \frac{a'}{\lambda} - \frac{c'}{L/D}$.	$\frac{b'}{\lambda}$.	$\frac{d'}{L/D}$.	$\phi = \frac{b'}{\lambda} + \frac{d'}{L/D}$.	$V = \sqrt{a\theta}$.	$P = \frac{\phi V^3}{375\theta}$.
.1	6.33	9.000	.157	8.843	.338	.271	.609	183.3	1130
.2	11.58	4.500	.086	4.414	.169	.148	.317	129.5	416
.3	15.93	3.000	.062	2.938	.113	.108	.221	105.6	237
.4	18.09	2.250	.055	2.195	.084	.095	.179	91.4	166
.5	18.93	1.800	.052	1.748	.068	.091	.159	81.5	131
.6	16.85	1.500	.059	1.441	.056	.102	.158	74.0	118
.7	15.65	1.286	.063	1.223	.048	.110	.158	68.2	109
.8	13.60	1.125	.073	1.052	.042	.126	.168	63.2	108
.9	11.61	1.000	.085	.915	.038	.148	.186	59.0	111
1.0	9.40	.900	.105	.795	.034	.183	.217	55.0	121

corresponding altitude is the ceiling, therefore the actual speed V′ is given by

$$V' = \frac{V}{\sqrt{\sigma}}.$$

Altitude.	σ.	V.	$V' = \frac{V}{\sqrt{\sigma}}$.
9,600	·747	55·2	63·9
10,700	·724	56·5	66·5
11,500	·704	57·9	69·1
12,500	·684	59·5	72·0
13,100	·669	61·1	74·8
13,700	·657	63·1	77·8
14,100	·650	65·3	81·0
14,100	·650	68·1	84·5
13,800	·655	71·2	88·0
13,300	·665	75·0	92·0
12,500	·684	79·6	96·3
10,500	·729	86·3	101·1
5,300	·866	100·0	107·5

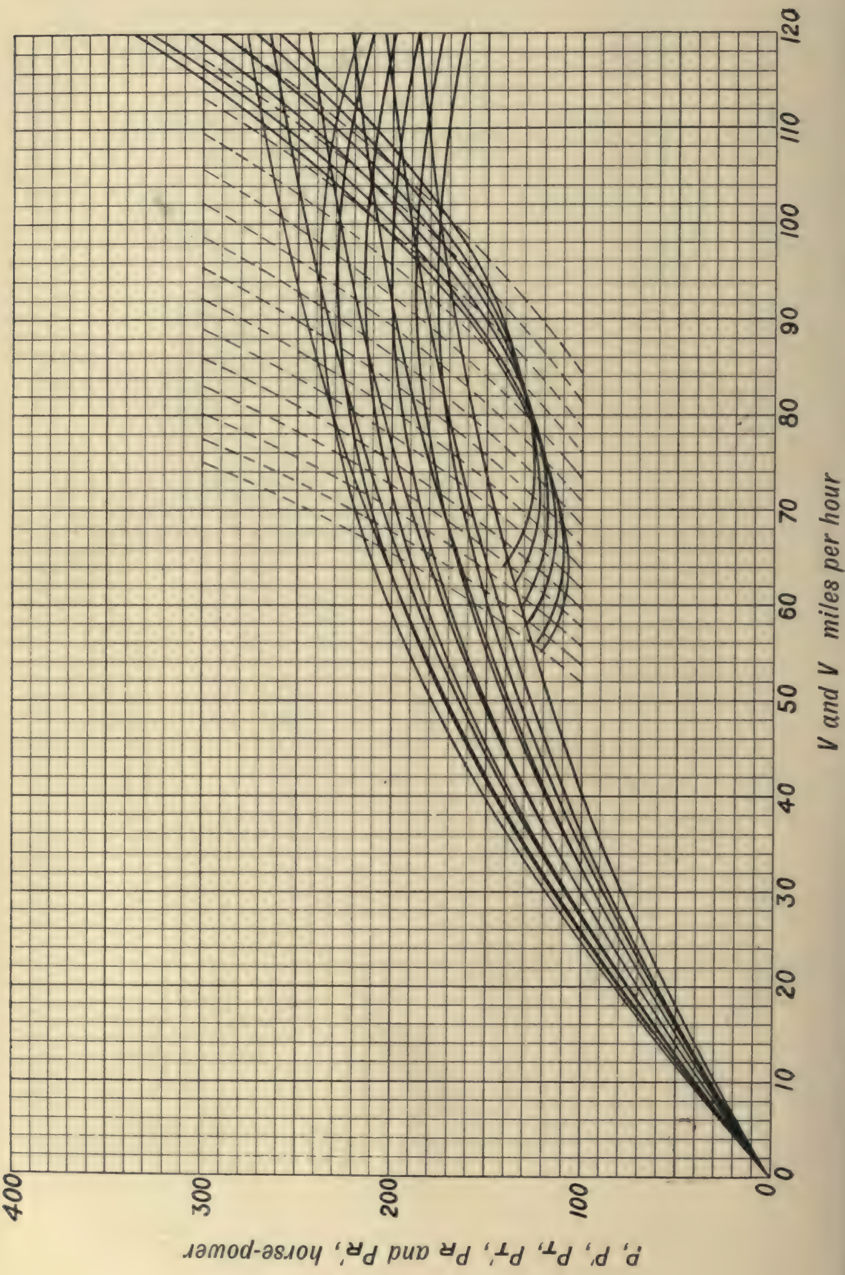
V′ is plotted against altitude of page 193, and what is obtained is really the curve of minimum speed under *full power* and the curve of top speed on *full torque:* in order to obtain the curves of *overall minimum speed*, and *overall maximum speed* we must supplement these curves by the curve of minimum speed *throttled*, and the curve of top speed on *full revolutions*.

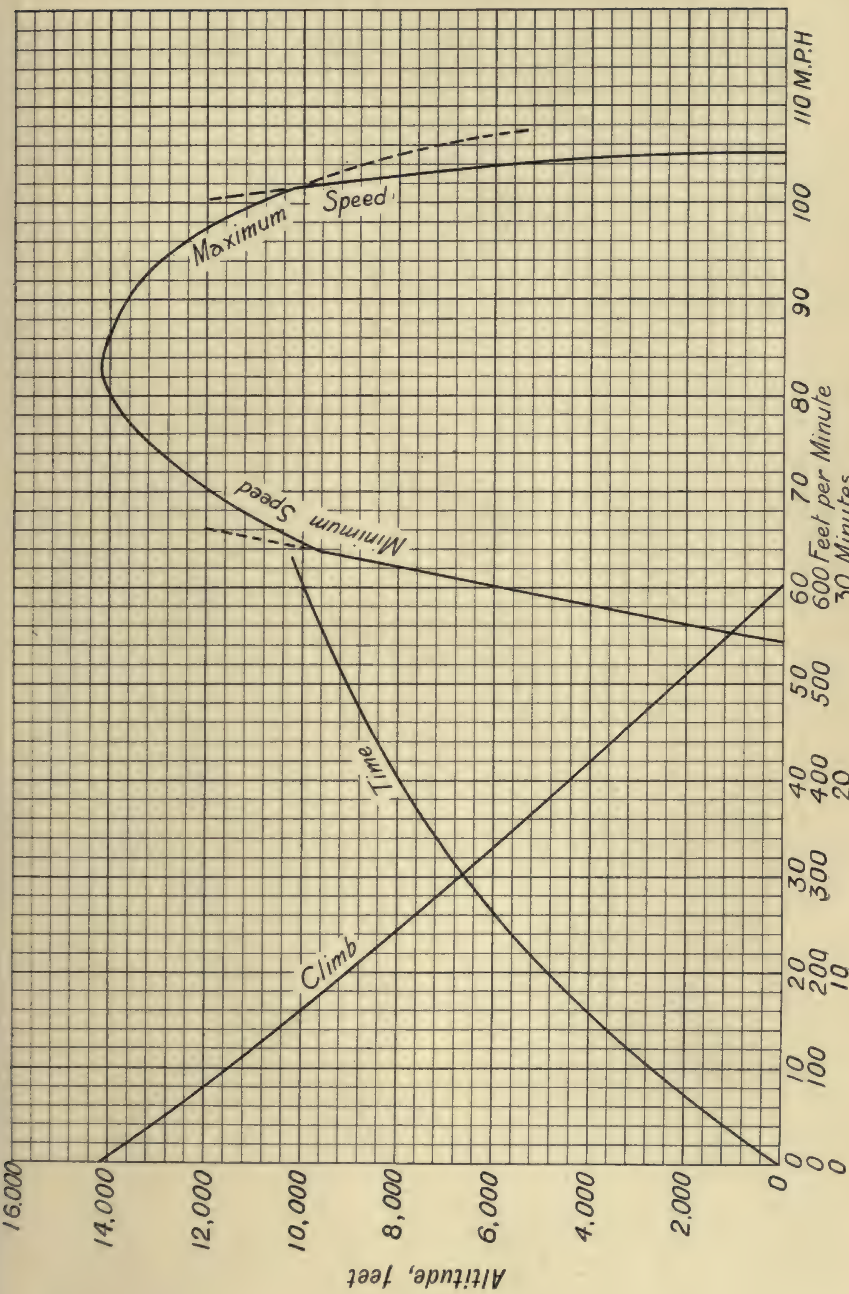
Meanwhile we observe that the maximum altitude attainable (what is ordinarily called the "ceiling") is 14,200 feet, and occurs at a true speed of 82 miles per hour.

Top Speed and Minimum Speed at Altitudes.— The minimum speed throttled in standard density air is 55·0 miles per hour. Therefore the minimum speeds throttled at altitudes of 2,000, 4,000, 6,000, 8,000, 10,000, and 12,000, where the values of σ are ·962, ·900, ·843, ·788, ·740, and ·694 respectively are given by the formula

$$V' = \frac{V}{\sqrt{\sigma}},$$

as 56·1, 58·0, 59·9, 62·0, 64·0, and 66·0 respectively.


V and V miles per hour
0
10
20
30
40
50
60
70
80
90
100
110
120
P, P', P_T, P_T', P_R and P_R', horse-power
0
100
200
300
400


Maximum Speed
Minimum Speed
Time
Climb
110 M.P.H
100
90
80
70
60
50
40
30
20
10
0
600 Feet per Minute
500
400
300
200
100
0
30 Minutes
20
10
0
16,000
14,000
12,000
10,000
8,000
6,000
4,000
2,000
0
Altitude, feet

These speeds are plotted against altitude on page 193, remembering that 800 feet is the altitude for standard density air. The old low speed curve already plotted is produced if necessary to cut the new curve, and the new curve is drawn in from this point of intersection to ground level: the part of the new curve above the intersection is not required, so it is shown dotted.

To find the top speed on full revolutions we have to find the performance curves at altitudes: we will do this at 2,000, 4,000, 6,000, and 8,000 feet, where the values of σ are ·962, ·900, ·843, and ·788 respectively.

For the machine performance curves at the altitudes we get:—

Altitude.		2,000.		4,000.		6,000.		8,000.	
V.	P.	V′.	P′.	V′.	P′.	V′.	P′.	V′.	P′.
183·3	1130	186·8	1152	193·2	1190	199·5	1230	206·3	1272
129·5	416	132·0	424	136·5	438	141·0	453	145·8	469
105·6	237	107·6	242	111·3	250	115·0	258	118·9	267
91·4	166	93·2	169	96·4	175	99·6	181	102·9	187
81·5	131	83·1	134	85·9	138	88·8	143	91·8	148
74·0	118	75·4	120	78·0	124	80·6	129	83·4	133
68·2	109	69·6	111	71·9	115	74·3	119	77·5	123
63·2	108	64·4	110	66·6	114	68·8	118	71·2	122
59·0	111	60·2	113	62·2	117	64·2	121	66·4	125
55·0	121	56·1	123	58·0	128	59·9	132	62·0	136

These are plotted on page 192.

For the constant revolutions propeller performance curves at the altitudes we get :—

Altitude.		2,000.	4,000.	6,000.	8,000.
$V' = V$.	P_R.	P_R'.	P_R'.	P_R'.	P_R'.
32	126	121	113	106	99
48	174	167	156	146	137
64	209	201	188	176	164
80	231	222	208	194	182
96	238	229	214	200	187
112	229	220	206	193	180

These also are plotted on page 192, and from the intersections of these curves with those of the previous table we find the top speed for full revolutions for standard density air, 2,000, 4,000, 6,000, and 8,000 feet to be 105·0, 105·0, 104·5, 103·6, and 102·8 miles per hour respectively.

On plotting these speeds on page 193 we see that we have not yet carried the investigation to high enough altitudes: we must therefore get our performance curves at 10,000 feet, where $\sigma = \cdot 740$.

For the machine performance curve we have velocities of 213·0, 150·5, 122·7, 106·2, 94·8, 86·0, 79·3, 73·5, 68·6, and 64·0, corresponding to horse-powers of 1313, 484, 276, 193, 152, 137, 127, 126, 129, and 141, and for the P_R' curve we have for velocites of 32, 48, 64, 80, 96, and 112, horse-powers of 93, 129, 154, 171, 176, and 169.

Plotting these on page 192 we get a full revolutions top speed at 10,000 feet of 101·6 miles per hour.

The curve of full revolutions top speed can now be plotted on page 193, but, of course, we only use the part of it below its intersection with the full torque top speed curve, while of the latter we only use the part above the intersection.

This completes the plotting of minimum and maximum speeds at all altitudes up to the ceiling.

Rate of Climb at all Altitudes.—For this purpose we require the curves of P_P', *i.e.* the curves of P_T' as well as those of P_R' which have already been found : we get :—

Altitude.		2,000.		4,000.		6,000.		8,000.		10,000.	
V.	P_T.	V′.	P_T'.	V′.	P_T'.	V′.	P_T'.	V′.	P_T'.	V′.	P_T'.
32	120	31·9	114	31·7	105	31·4	96	31·2	87	30·9	80
48	166	47·8	158	47·5	146	47·1	132	46·8	121	46·4	110
64	202	63·8	192	63·4	177	62·8	161	62·4	147	61·8	134
80	231	79·7	220	79·2	203	78·6	184	77·9	168	77·2	154
96	253	95·6	241	95·0	222	94·2	202	93·5	184	92·7	168
112	270	111·5	257	110·8	237	110·0	215	109·0	197	108·1	180

since for these altitudes the formula $\sigma_1 = \dfrac{\sigma - p}{q}$ gives $\sigma_1 = ·955$, ·882, ·813, ·748, and ·690. The above values of P_T' are also plotted on page 192.

Using the method of the First Approximation we observe that the rate of climb in feet per minute is given by

$$C = 33{,}000\frac{P_P - P}{W}.$$

We will therefore save time by scaling off the *maximum value of* $P_P - P$ *only* with dividers from the curves of page 192. Hence we get :—

Altitude.	$P_P - P$.	$C = 5{\cdot}5\,(P_P - P)$.
Standard.	103	567
2,000	93	511
4,000	78	429
6,000	60	330
8,000	43	236
10,000	30	165

These are plotted on page 193.

Times to all Altitudes.—The plotting of rate of climb on altitude is (at least up to 10,500 feet) a close approximation to the straight line which cuts the ground level line at 600 feet per minute and the "no climb" line at 13,600 feet.

Therefore the time t in minutes to the height a (up to 10,500 feet at least) is given by

$$t = \frac{2{\cdot}303 a_1}{c} \log_{10}\left(\frac{a_1}{a_1 - a}\right)$$

where $c = 600$ and $a_1 = 13{,}600$.

a.	$13{,}600 - a$.	$\frac{13{,}600}{13{,}600 - a}$.	$\log\left(\frac{13{,}600}{13{,}600 - a}\right)$.	$t = 52{\cdot}2 \log\left(\frac{13{,}600}{13{,}600 - a}\right)$.
2,000	11,600	1·172	·06893	3·6
4,000	9,600	1·417	·1514	7·9
6,000	7,600	1·790	·2529	13·2
8,000	5,600	2·430	·3856	20·1
10,000	3,600	3·780	·5775	30·1

These times are plotted on page 193.

Best Altitude for Cruising Against a Twenty Mile Wind.—First we must find the relative consumption a and also the quantity $\frac{a}{V' - v}$ for a range of speeds and altitudes.

$$p = {\cdot}161.$$

Standard. $\sigma = 1{\cdot}000$; $p = {\cdot}161$; $(\sigma - p) = {\cdot}839$.

V.	V_T.	V_R.	$(\sigma - p)\left(\frac{V}{V_T}\right)^2$.	$p + (\sigma - p)\left(\frac{V}{V_T}\right)^2$.	$\alpha = \left[p + (\sigma - p)\left(\frac{V}{V_T}\right)^2\right]\frac{V}{V_R}$.	V − 20.	$\frac{\alpha}{V - 20}$.
55·2	66·1	67·1	·585	·746	·614	35·2	·0174
56·5	69·0	69·8	·562	·723	·586	36·5	·0160
57·9	72·0	72·7	·543	·704	·561	37·9	·0148
59·5	75·4	75·6	·522	·683	·538	39·5	·0136
61·1	78·5	78·7	·508	·669	·520	41·1	·0127
63·1	82·1	82·0	·495	·656	·505	43·1	·0117
65·3	85·5	85·0	·490	·651	·500	45·3	·0110
68·1	89·2	88·3	·489	·650	·501	48·1	·0104
71·2	92·8	91·4	·493	·654	·510	51·2	·0100
75·0	96·8	94·6	·504	·665	·527	55·0	·0096
79·6	100·8	97·9	·523	·684	·556	59·6	·0093
86·3	104·9	101·3	·569	·730	·622	66·3	·0094
100·0	109·0	104·5	·706	·867	·830	80·0	·0104

2,000 feet. $\sigma = {\cdot}962$; $p = {\cdot}161$; $(\sigma - p) = {\cdot}801$.

V'.	V_T'.	V_R'.	$(\sigma - p)\left(\frac{V'}{V_T'}\right)^2$.	$p + (\sigma - p)\left(\frac{V'}{V_T'}\right)^2$.	$\alpha = \left[p + (\sigma - p)\left(\frac{V'}{V_T'}\right)^2\right]\frac{V'}{V_R'}$.	$V' - 20$.	$\frac{\alpha}{V' - 20}$.
57·0	67·8	68·7	·566	·727	·603	37·0	·0163
58·5	70·8	71·5	·547	·708	·579	38·5	·0150
60·2	74·0	74·6	·530	·691	·558	40·2	·0139
61·8	77·0	77·4	·516	·677	·541	41·8	·0135
63·7	80·5	80·6	·502	·663	·524	43·7	·0120
65·7	83·7	83·7	·494	·655	·514	45·7	·0112
68·4	87·5	87·0	·490	·651	·512	48·4	·0106
71·2	91·0	90·1	·490	·651	·514	51·2	·0100
75·0	95·0	93·4	·499	·660	·530	55·0	·0096
79·0	98·8	96·6	·512	·673	·550	59·0	·0093
85·0	103·0	100·0	·545	·706	·600	65·0	·0092
94·6	107·0	103·2	·626	·787	·722	74·6	·0097

4,000 feet. $\sigma = \cdot 900$; $p = \cdot 161$; $(\sigma - p) = \cdot 739$.

V'.	V_T'.	V_R'.	$(\sigma - p)\left(\frac{V'}{V_T'}\right)^2$.	$p + (\sigma - p)\left(\frac{V'}{V_T'}\right)^2$.	$\alpha = \left[p + (\sigma - p)\left(\frac{V'}{V_T'}\right)^2\right]\frac{V'}{V_R'}$.	$V' - 20$.	$\frac{\alpha}{V' - 20}$.
58·2	65·5	66·8	·584	·745	·649	38·2	·0170
59·6	68·6	69·8	·558	·719	·614	39·6	·0155
61·2	71·5	72·7	·542	·703	·600	41·2	·0146
62·9	74·6	75·5	·525	·686	·571	42·9	·0133
64·6	78·0	78·8	·507	·668	·548	44·6	·0123
66·6	81·3	81·9	·496	·657	·534	46·6	·0115
69·0	84·8	85·1	·490	·651	·528	49·0	·0108
71·6	88·3	88·1	·486	·647	·526	51·6	·0102
75·0	92·0	91·3	·491	·652	·536	55·0	·0097
79·0	95·9	94·6	·502	·663	·554	59·0	·0094
83·4	99·8	97·9	·516	·677	·576	63·4	·0091
90·2	104·0	101·2	·557	·718	·640	70·2	·0091
104·5	108·2	104·5	·690	·851	·851	84·5	·0101

6,000 feet. $\sigma = \cdot843$; $p = \cdot161$; $(\sigma - p) = \cdot682$.

V'.	V_T'.	V_R'.	$(\sigma - p)\left(\frac{V'}{V_T'}\right)^2$.	$p + (\sigma - p)\left(\frac{V'}{V_T'}\right)^2$.	$\alpha = \left[p + (\sigma - p)\left(\frac{V'}{V_T'}\right)^2\right]\frac{V'}{V_R'}$	$V' - 20$.	$\frac{\alpha}{V' - 20}$.
60·6	66·0	67·9	·575	·736	·657	40·6	·0162
62·2	68·7	70·8	·560	·721	·634	42·2	·0150
63·7	71·8	73·6	·537	·698	·604	43·7	·0138
65·5	75·1	76·6	·519	·680	·582	45·5	·0128
67·4	78·3	79·7	·505	·666	·563	47·4	·0119
69·9	81·9	82·9	·497	·658	·555	49·9	·0111
72·3	85·3	86·0	·490	·651	·547	52·3	·0105
75·2	88·9	89·2	·488	·649	·547	55·2	·0099
79·0	92·4	92·4	·499	·660	·564	59·0	·0096
83·4	96·4	95·7	·511	·672	·586	63·4	·0092
88·4	100·1	99·0	·532	·693	·619	68·4	·0091
98·0	104·5	102·4	·600	·761	·729	78·0	·0093

8,000 feet. $\sigma = \cdot788$; $p = \cdot161$; $(\sigma - p) = \cdot627$.

V'.	V_T'.	V_R'.	$(\sigma - p)\left(\frac{V'}{V_T'}\right)^2$.	$p + (\sigma - p)\left(\frac{V'}{V_T'}\right)^2$.	$a = \left[p + (\sigma - p)\left(\frac{V'}{V_T'}\right)^2\right]\frac{V'}{V_R'}$.	$V' - 20$.	$\frac{a}{V' - 20}$.
63·2	66·3	68·8	·570	·731	·672	43·2	·0156
64·7	69·3	71·5	·547	·708	·641	44·7	·0143
66·6	72·2	74·8	·534	·695	·619	46·6	·0133
68·5	75·4	77·6	·517	·678	·598	48·5	·0123
70·6	78·7	81·0	·505	·666	·581	50·6	·0115
73·0	82·1	84·0	·495	·656	·570	53·0	·0108
76·0	85·6	87·2	·495	·656	·572	56·0	·0102
79·2	89·2	90·4	·495	·656	·575	59·2	·0097
83·4	93·0	93·6	·505	·666	·593	63·4	·0094
87·6	97·0	97·0	·511	·672	·607	67·6	·0090
94·4	100·8	100·2	·550	·711	·670	74·4	·0090

10,000 feet. $\sigma = \cdot740$; $p = \cdot161$; $(\sigma - p) = \cdot579$.

V'.	V_T'.	V_R'.	$(\sigma - p)\left(\frac{V'}{V_T'}\right)^2$.	$p + (\sigma - p)\left(\frac{V'}{V_T'}\right)^2$.	$a = \left[p + (\sigma - p)\left(\frac{V'}{V_T'}\right)^2\right]\frac{V'}{V_R'}$.	$V' - 20$.	$\frac{a}{V' - 20}$.
64·4	64·0	67·2	·586	·747	·716	44·4	·0161
65·8	66·7	70·0	·563	·724	·681	45·8	·0149
67·6	70·0	73·0	·540	·701	·649	47·6	·0136
69·2	72·8	76·0	·523	·684	·623	49·2	·0127
71·4	76·1	79·0	·510	·671	·606	51·4	·0118
73·6	79·4	82·0	·497	·658	·591	53·6	·0110
76·4	82·8	85·2	·493	·654	·586	56·4	·0104
79·4	86·4	88·5	·489	·650	·583	59·4	·0098
83·2	90·0	91·6	·495	·656	·596	63·2	·0094
87·2	93·6	94·8	·503	·664	·611	67·2	·0091
92·6	97·8	98·2	·519	·680	·641	72·6	·0088
100·5	101·5	101·3	·567	·728	·722	80·5	·0090

On plotting $\frac{a}{V' - 20}$ against V′ we get at standard, 2000, 4000, 6000, 8000, and 10,000 feet minima of ·0093, ·0092, ·00905, ·0091, ·00895, and ·0088. On plotting these minima against altitude we do not get a minimum, as the value is still getting smaller even at 10,000 feet.

We have carried the investigation far enough, however, to show that with a head wind of even 20 miles per hour it pays to fly high.

In the course of the above plottings we find that the best cruising speeds at the respective altitudes are about 82, 86, 88, 90, 91, and 94 miles per hour respectively. The curves are not reproduced as they are of no special interest.

Cruising Range against a Twenty Mile Head Wind at 6000 Feet.—We see from the above that it will pay us to use $V_0' = 90$: then by interpolating on the 6000 foot table above we get $V_T' = 101$ and $V_R' = 99{\cdot}7$.

Also $W_0 = 6000$, $W = 3600$, $p = {\cdot}161$ and we can take $\Delta = 193$, while at 6000 feet $\sigma = {\cdot}843$.

$$\therefore\ a = \frac{193 \times {\cdot}682 \times 90^2}{6000 \times 99{\cdot}7 \times 101^2} = {\cdot}0001748$$

and
$$w = 3600 + \frac{{\cdot}161 \times 101^2 \times 6000}{{\cdot}682 \times 90^2} = 5385$$

and
$$w_0 = 6000\left[1 + \frac{{\cdot}161 \times 101^2}{{\cdot}682 \times 90^2}\right] = 7785.$$

$$\therefore\ x = \frac{2{\cdot}303}{{\cdot}0001748} \log_{10}\left(\frac{7785}{5385}\right) = 2110$$

and
$$b = \sqrt{\frac{{\cdot}161 \times 101^2 \times 6000}{{\cdot}682 \times 90^2}} = 42{\cdot}25$$

$$\therefore\ t = \frac{{\cdot}0349\sqrt{6000}}{42{\cdot}25 \times {\cdot}0001748 \times 90} \tan^{-1}\left\{\frac{42{\cdot}25(\sqrt{6000} - \sqrt{3600})}{1785 + \sqrt{6000 \times 3600}}\right\} = 26{\cdot}65.$$

Therefore the cruising range in miles is

$$x - v't = 2110 - 20 \times 26{\cdot}65 = 1577 \text{ miles.}$$

This figure, together with the other performance particulars already plotted on page 193, are all that are required in an ordinary case. Any other figures that may happen to be wanted can be got as explained in the earlier chapters.

INDEX.

NOTE.—Page numbers in Part I. are printed in ordinary type, page numbers in Part II. in black type, and page numbers in Part III. in italics.